Time-Frequency Analysis for Multi-Channel Color Pattern Recognition

Reiner Lenz
Department of Science and Technology, Linköping University
SE-60174 Norrköping, Sweden

Javier Hernández-Andrés
Department of Optics, Sciences Faculty, University of Granada,
18071 Granada, Spain

Abstract

Today almost all color images are captured by RGB cameras. They describe color information by three measurements only. This is too restrictive for many applications and alternative, multichannel color description techniques have thus received a lot of attention recently. In this paper we will describe some tools from time-frequency analysis and study if and how they can be used for multichannel color signal processing. We will mainly use these tools to study if they can be used to investigate problems involving scenes illuminated by different illumination sources.

Introduction

Traditional color imaging devices are almost all based on three channels. This is too restrictive for many applications and has lead to a number of new imaging technologies that capture more than three channels per pixel. Recently a new digital camera has reached the consumer market that is based on four channel color measurements. The evaluation of these new color images requires new methods and models and in this paper we will introduce time-frequency analysis as a toolbox that provides many techniques that could be useful in the analysis of multispectral color information. We will introduce some basic tools from time-frequency-analysis (TFA) and describe how they can be used to characterize different color signals. We will illustrate their usage by solving some pattern recognition problems involving the influence of different types of illumination sources.

In this paper we will mainly focus on the pattern recognition applications of TFA and only sketch the basic facts from the general theory. A detailed description of time-frequency methods can be found in the literature (see [1] for an textbook introduction, [2] for a recent review or [3] in the current volume).

Time-Frequency Analysis

Color signals are usually described as functions defined on the interval of wavelengths of interest (like 350nm to 800nm). This description is complete in the sense that all physics-related properties of this single color signal can be derived from this description. This description is however redundant and many properties are not directly visible. Therefore other representations were developed and used in multichannel color processing. Examples of such representations are principal component analysis or Fourier transform based methods. Here we will describe time-frequency methods and illustrate their application in pattern recognition. As an example consider the three illumination spectra shown in Figure 1. Two of the lamps are characterized by the location of the distributions, the lamp concentrated in the short wavelength range is a blue light, the lamp concentrated in the long wavelength range is a red light. Both of these lamps have in common that their distributions are very smooth, they consist of a single Gaussian shaped function. The property that discriminates the third lamp from the other two is its form. This spectral distribution consists of a superposition of several narrow-band spectral distributions. The difference between the first two lamps and the third lamp is therefore easier to describe in terms of their Fourier transforms. The first two lamps are low-frequency distributions whereas the third one contains substantial high-frequency contributions. Time-frequency methods try to combine both, the position and the frequency aspects of signals into a single representation.

Time-frequency methods were first developed in the field of quantum mechanics in the thirties and later they were applied to study problems in one-dimensional signal processing, especially in radar and sound processing. Here we view the spectral distribution in a pixel as a one-dimensional signal and analyze it with the Wigner (also called the Wigner-Ville) distribution, the oldest and most

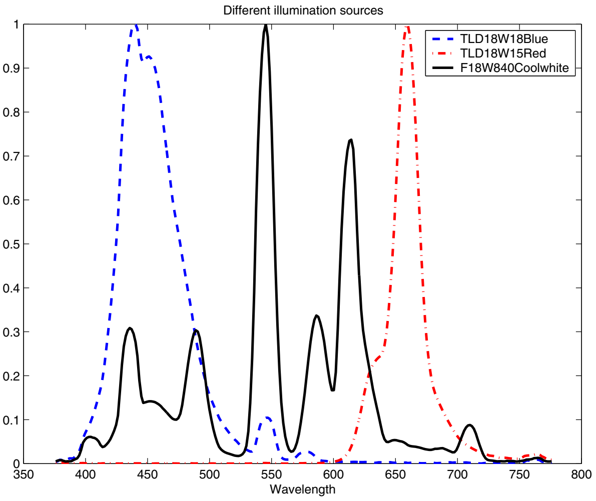

Figure 1: *Illumination spectra for three lamps*

important time-frequency method. For a color signal $s(\lambda)$ this distribution is defined as:

$$W_s(\lambda, \xi) = \int s\left(\lambda + \frac{\tau}{2}\right) s\left(\lambda - \frac{\tau}{2}\right) \mathrm{e}^{-i\tau\xi}\, d\tau \qquad (1)$$

From the definition we can see the following important properties: (1) only the signal is involved in the definition and (2) the transform is quadratic and not linear. The first property is certainly an advantage since it is not necessary to select a basis as in most other multi-channel approaches. The second property however complicates the application of the Wigner distribution.

For the rest of the paper we will focus on one of the more intuitive interpretations of the Wigner distribution as a time-frequency energy distribution: the value of W_s at the point (λ_0, ξ_0) is the probability that there was a variation of frequency ξ_0 around the wavelength λ_0. Among the many properties of the Wigner distribution we mention that it has real values, that for most signals it assumes negative values and that it satisfies the marginals (see [3]). We also mention that for many applications it is more convenient to investigate the analytical signal $z(\lambda) = s(\lambda) + i(Hs)(\lambda)$ where Hs is the Hilbert transform of s (also this is described in [3]).

We want to use the Wigner distribution to solve pattern recognition problems and therefore we need to characterize properties of the Wigner distribution with a few characteristic measurements. If we think of the Wigner distribution as a probability distribution we can first characterize the complexity of a distribution with the entropy. Since the ordinary (Shannon) entropy $\int p(\omega) \ln p(\omega)\, d\omega$ is only defined for positive-valued functions we have to use an alternative type of entropy. Here we measure the complexity of a time-frequency distribution W_s by the Renyi information (usually we set $\alpha = 3$):

$$R^\alpha(W_s) = \frac{1}{1-\alpha} \log_2 \left(\int \int W_s^\alpha(\lambda, \xi)\, d\lambda d\xi \right) \qquad (2)$$

For the three illumination sources shown in Figure 1 we get the Renyi information listed in Table 1. We see that the red and the blue lamps have comparable values whereas the Coolwhite lamp has a substantially higher entropy. This indicates that Coolwhite is less concentrated than the red and blue lamps.

Illumination	Renyi information
TLD18W18 Blue	8.6991
TLD18W15 Red	8.7599
F18W840 Coolwhite	9.6046

Table 1: *Renyi information of illumination sources*

Another set of useful measures to characterize probability distributions are moments, such as the mean or the variance. For the pure moments (i.e. moments in the wavelength or the frequency variables alone) we get:

$$\mu_{n0} = \int \lambda^n W_s(\lambda, \xi)\, d\lambda d\xi = \int \lambda^n |s(\lambda)|^2\, d\lambda$$
$$\mu_{0m} = \int \xi^m W_s(\lambda, \xi)\, d\lambda d\xi = \int \xi^m |S(\xi)|^2\, d\xi \quad (3)$$

Mixed moments are computed as

$$\mu_{nm} = \int \lambda^n \xi^m W_s(\lambda, \xi)\, d\lambda d\xi \qquad (4)$$

Moments of order up to two will later be used in the experiments.

TFA and Stochastic Processes

Up to now we considered single color signals and their Time-Frequency-Analysis. In pattern recognition we are however mainly interested in sets of color signals. Often these color spectra can be considered as results of stochastic processes. In this case we write $s(\lambda, \omega)$ with a stochastic variable ω instead of $s(\lambda)$. We use E to denote the mean or the expectation operator, i.e. integration over ω.

For the mean of the Wigner distribution of a stochastic process we get

$$
\begin{aligned}
&\overline{W}_s(\lambda, \xi) = \\
&= E\left[\int s\left(\lambda + \frac{\tau}{2}, \omega\right) s\left(\lambda - \frac{\tau}{2}, \omega\right) \mathrm{e}^{-i\tau\xi}\, d\tau\right] \\
&= \int E\left[s\left(\lambda + \frac{\tau}{2}, \omega\right) s\left(\lambda - \frac{\tau}{2}, \omega\right)\right] \mathrm{e}^{-i\tau\xi}\, d\tau \\
&= \int R\left(\lambda + \frac{\tau}{2}, \lambda - \frac{\tau}{2}\right) \mathrm{e}^{-i\tau\xi}\, d\tau \qquad (5)
\end{aligned}
$$

where $R(\lambda_1, \lambda_2) = E[s(\lambda_1) s(\lambda_2)]$ is the correlation function of the stochastic process. The function $\overline{W}_s$ is known as the Wigner-Ville-Spectrum of the stochastic process.

Next we note that the correlation function defines an integral operator via the definition

$$
f_R(\lambda_2) = \int R(\lambda_1, \lambda_2) f(\lambda_1)\ d\lambda_1
$$

This is a positive definite operator and has therefore eigenfunctions b_k with positive eigenvalues μ_k that span the whole function space. Therefore we can write the correlation function as:

$$
R(\lambda_1, \lambda_2) = \sum_k \mu_k b_k(\lambda_2) b_k(\lambda_1)
$$

For more details see [4] (Ch. 97 and 98).

If we insert this in the equation for the Wigner-Ville-Spectrum then we get:

$$
\begin{aligned}
&\overline{W}_s(\lambda, \xi) = \\
&= \int R\left(\lambda + \frac{\tau}{2}, \lambda - \frac{\tau}{2}\right) \mathrm{e}^{-i\tau\xi}\, d\tau \\
&= \int \sum_k \mu_k b_k\left(\lambda + \frac{\tau}{2}\right) b_k\left(\lambda - \frac{\tau}{2}\right) \mathrm{e}^{-i\tau\xi}\, d\tau \\
&= \sum_k \mu_k W_k(\lambda, \xi) \qquad (6)
\end{aligned}
$$

where the W_k are the Wigner-distributions of the eigenfunctions b_k of the correlation operator: the Wigner-Ville-spectrum is the weighted sum of the Wigner-Distributions of the eigenfunctions of the correlation operator with the weights equal to the eigenvalues.

If the stochastic process consists of positive functions then it follows from the Krein-Rutman theory (see [5] (pp. 2129)) that the first eigenfunction is positive. This implies that the first eigenfunction is very similar to the mean function. Since the first eigenvalue is also in most cases very dominant we find that in most cases the Wigner-Ville-Distribution of the process is very similar to the Wigner-Distribution of the mean function. Under our simplified reflection model we see also that the mean of the reflected spectra is the point-wise product of the mean of the reflection spectra and the illumination spectrum. Under the condition that the mean reflectance spectrum is almost a constant we see therefore that the Wigner-Ville-Spectrum should be very similar to the Wigner Distribution of the illumination spectrum.

Experiments

In the first experiment we illustrate the similarity between the Wigner-Ville-Spectrum and the Wigner-Distribution of the illumination source. For this purpose we use 219 reflectance spectra from natural objects and 15 illumination spectra (artificial light sources of different types of lamps). We then compute the Wigner-Ville spectrum of the stochastic process defined by the reflectance spectra and a given, fixed illumination source. We thus have 219 realizations of a given process. Figure 2 shows the Wigner-Ville-Spectrum of the process when we use the illumination source F18W840 Coolwhite and the 219 spectra and in Figure 3 we show the Wigner-Distribution of the light source (Here we do not use the analytical signal).

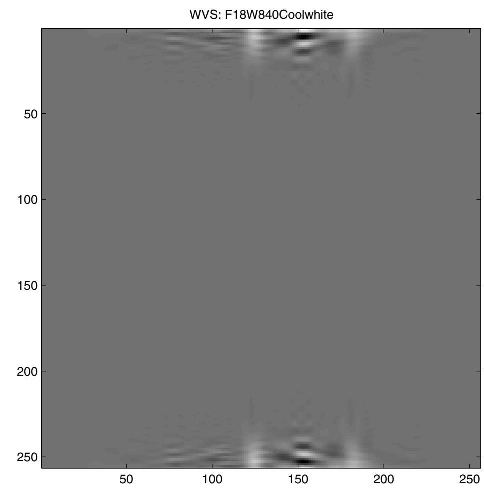

Figure 2: *Wigner-Ville-Spectrum*

In the next series of experiments we investigate if the tools from time-frequency analysis can be used to automatically classify color signals depending on their underlying illumination sources. The experimental setup was as follows: First we select a larger set of reflectance spectra and a number of illumination sources and their spectral distributions. We assume that the spectrum of the light reflected from an object point is the pointwise product of the reflection spectrum and the illumination spectrum. From the resulting spectra we eliminate first intensity variations by normalizing the spectra to norm one. The problem is

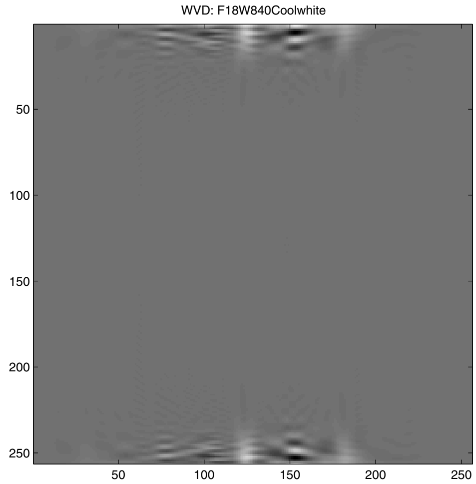

Figure 3: *Wigner-Distribution of the Illumination*

Figure 5: *Original Image Scene1*

now if it is possible to characterize the shape of these normalized spectra with the help of time-frequency measurements. We thus characterize color signals by their "chromaticity" properties and want to classify them depending on their underlying illumination source.

In the following figures we illustrate some of our experiments. Here we use the two scenes shown in Figures 4 and 5 (they are described in [6]) and the two (very similar) illumination spectra shown in Figure 6. We selected these images since they have very different color characteristics.

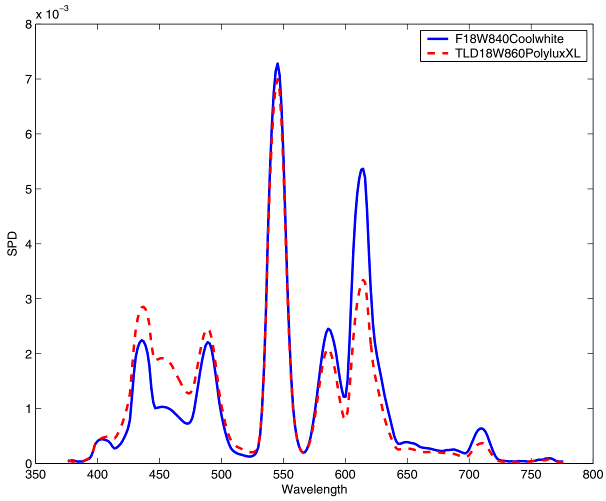

Figure 6: *Two illumination spectra*

Figure 4: *Original Image Scene5*

We first divided the images into rectangular regions and then simulated the interaction of the reflectance spectra with the different light sources by multiplying the reflection spectra with the illumination spectra. We used more illumination spectra but used only the two illumination spectra in Figure 6 to test the classification performance. The feature extraction step in the experiment was as follows: From the resulting color signal we computed first its norm and then we normalized the color signal to norm one. The remaining color information was thus independent of the intensity (in the L^2 sense). From the normalized color signal we computed the analytical signal and its Wigner distribution. Then we computed the Renyi entropy, the two first order and the three second order moments of the Wigner distribution. The result is a six-dimensional measurement or feature vector in each image pixel. Then we trained a classifier by selecting two illumination sources and a random set of 1000 pixels from each class. These pixels were used to train a classifier. Here we used the 'treefit' method from the Matlab statistics toolbox to fit a tree-based model. Then we apply the learned classifier to all pixels in the image. To visualize the result we mapped all classification results to black and white (black characterizing class one and white the complementary class). Those pixels in the image that were illuminated by a source different from the two selected sources where set to a common gray value (either black or white). In Figures 7 and 8 we see which patches in Scene 5 where classified as being illuminated by TLD18W860PolyluxXL (in white) and by F18W840Coolwhite (in black). Only the blocks in the diagonals participated in the test, the remaining pixels were illuminated by the other sources. Figure 9 shows one of the results of the same experiment but now with Scene 1.

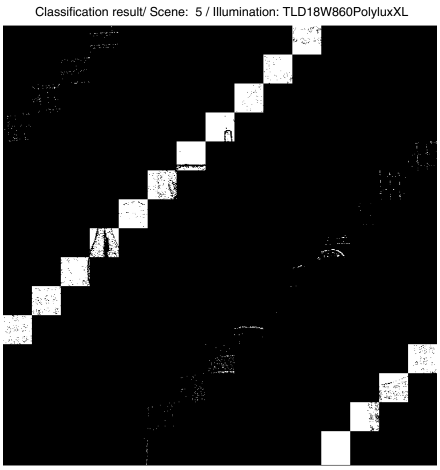

Figure 7: *Points classified as illuminated by TLD18W860PolyluxXL*

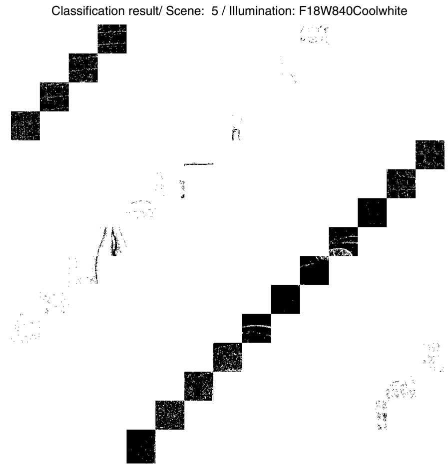

Figure 8: *Points classified as illuminated by F18W840Coolwhite*

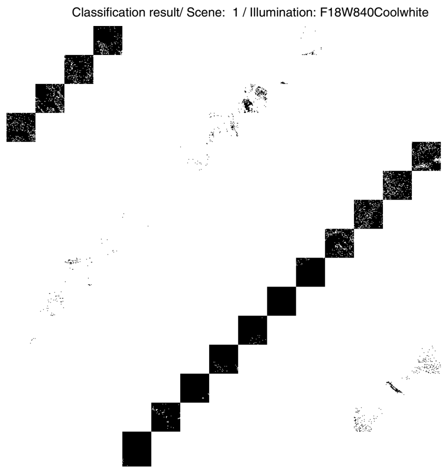

Figure 9: *Points classified as illuminated by by F18W840Coolwhite*

In the next experiment we selected four spectra as illumination sources (see Figure 10). From the pixels illuminated by a source we select randomly 500 points and learn a regression classifier (as implemented in the Matlab Statitics Toolbox) from the known 4×500 samples. Then we apply the classifier to all pixels illuminated by the selected sources. We then compute how the pixels under a given illumination were classified by the classifier. In Figure 11 we see the classification results obtained. The curves in this figure show the probability distributions of the classification results for the different illumination sources. They show that the pixels illuminated by the source TLD18W35White were almost all correctly classified whereas the pixels illuminated by the other three sources had a wider spreading. The true class labels (which are of course arbitrary) were 2,4,6 and 8. This example shows that using only standard tools from the theory of time-frequency analysis and a standard classification method from Matlab can produce useful results to solve difficult color image processing problems.

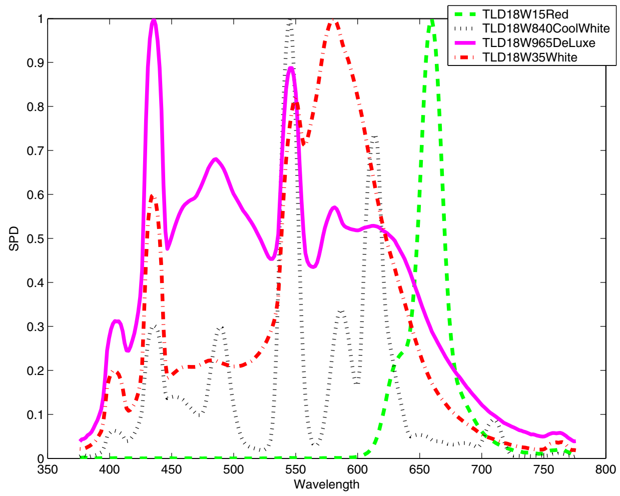

Figure 10: *Four normalized illumination spectra*

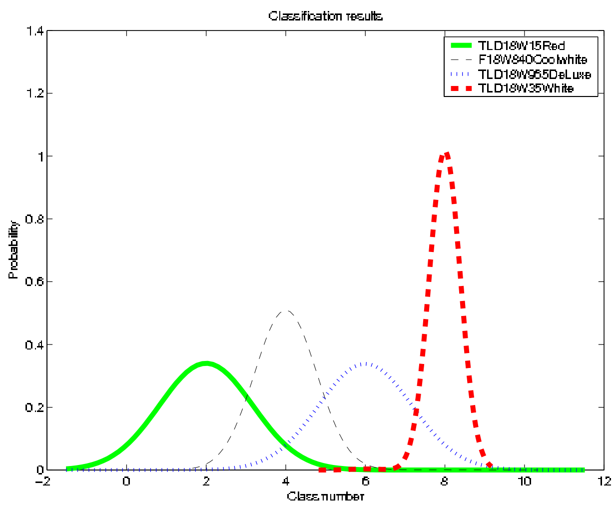

Figure 11: *Classification results for four illumination spectra*

Conclusions

In this paper we showed that Time-Frequency-Distributions (especially the Wigner-Distribution) provide a description of color signals that combine attractive features from both the wavelength and the Fourier Transform description. We also generalized the application of some Time-Frequency-Analysis methods from the application to single color signals to stochastic processes of color signals. We combined Time-Frequency-Analysis with the Krein-Rutman theory of positive processes to analyze the Wigner-Ville spectrum of stochastic processes of color signals. In the experimental part of the paper we used simple probabilistic characterizations of color signals (like the entropy and moments) to classify color signals generated by different illumination sources. We showed that even relatively similar illumination sources could be classified correctly if they interacted with a larger number of reflection spectra. We also did similar experiments in which we used measured daylight, skylight and twilight spectra and simulated their interaction with real scenes. The results obtained in these experiments are similar to the results presented above.

These results are only thought as examples that should demonstrate how some basic tools from time-frequency analysis can be used to investigate multispectral pattern recognition problems. We do not claim that this is an optimal solution (further studies will probably lead to much better results) but we showed that even these elementary tools can produce useful results.

Acknowledgements

The reflection and illumination spectra were provided by the Color Group at the Dept. Comp. Science Joensuu University of Technology, Finland (See also `http://cs.joensuu.fi/~spectral`). We used the Matlab Toolbox available at `http://crttsn.univ-nantes.fr/~auger/tftb.html`. The multi-spectral images used are described in [6]. The support of the Swedish Research Council is gratefully acknowledged.

References

[1] L. Cohen, *Time Frequency Analysis*. Signal Processing Series, Englewood Cliffs, NJ, USA": Prentice Hall, 1995.

[2] G. Matz and F. Hlawatsch, "Wigner distributions (nearly) everywhere: time frequency analysis of signals, systems, random processes, signal spaces, and frames," *Signal Processing*, vol. 83, no. 7, pp. 1355–1378, 2003.

[3] R. Lenz, "Time-frequency analysis of color spectra," in *2nd European Conf. Color in Graphics, Imaging and Vision*, 2004. (accepted).

[4] F. Riesz and B. S. Nagy, *Functional Analysis*. Dover Publications Inc., 1990.

[5] N. Dunford and J. T. Schwartz, *Part III, Spectral Operators*. New York: Interscience Publishers, 1988.

[6] S. M. C. Nascimento, F. P. Ferreira, and D. H. Foster, "Statistics of spatial cone-excitation ratios in natural scenes," *JOSAA*, vol. 19, no. 8, pp. 1484–1490, 2002).