Optimal Spectral Sensitivities of a Color Image Acquisition Device in the Presence of Noise

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Abstract

One of the most important aspects of a set of color image sensors is to capture colorimetric information about an object being imaged. The estimation accuracy of the colorimetric information depends not only on the spectral sensitivities of the sensors but also on the noise present in the color image acquisition device. In this paper, for the technical tractability, the problem on the design of an optimal set of spectral sensitivities with Gaussian distribution functions is addressed based on a colorimetric evaluation model. It is shown that the shapes of the optimal spectral sensitivities change to linearly independent sensitivity curves with a decrease in signal-to-noise ratio (SNR) to suppress the noise effects and that optimization must be done at a specific noise level of a device.

Introduction

With the recent progress in the color management system, a color image acquisition device is required to capture accurate colorimetric information of an object being imaged. Several models have been proposed to evaluate the colorimetric performance of a set of color sensors[1-3] and the optimization of it has been carried out as an optimization problem to minimize some criteria.[4,5]

In the approaches to acquire colorimetric information, the criteria used for an optimal set of sensitivities were the minimization of the mean square errors (MSE) in a normalized tristimulus space[4,5] and in a perceptual color space[5]. The optimizations were performed in the presence of noise. It was confirmed that an increased number of sensors over the four is not useful to acquire colorimetric information at low SNR's[4,5] and that the optimization of the filter shapes at a certain SNR is good enough since the changes in the shape of the nonnegative filters for a change in SNR are not dramatic.[4] Since the criteria used for the optimization was the minimization of the MSE in a normalized tristimulus space, the intuitive insight into the results on the optimization was not clear.[4,5] The shapes of the spectral transmittance of the nonnegative filters have complicated shapes.[4] Although the manufacturability of the optical filters with the complicated characteristics of the transmittance is demonstrated,[6] but optical interference filters usually manufactured and available in commercial are Gaussian filters.[7] Therefore for the technical tractability, the optimization of a set of spectral sensitivities with a Gaussian distribution functions is addressed in this paper. It was found that a set of Gaussian sensitivities optimized at low noise level is very sensitive to noise and that an increase in the number of channels is not always effective to capture colorimetric information accurately since the noise's contribution to the MSE increases with the sensor number at low SNR's. It is shown that the proposed colorimetric evaluation model[2] can explain the findings in the above and that a set of Gaussian filters must be optimized for a specific SNR.

Colorimetric Evaluation Model

A sensor response vector from a set of color sensors for an object with a $N \times 1$ (N represents the sampling number) spectral reflectance vector $\mathbf{r}$ can be expressed by

$$\mathbf{p} = \mathrm{SL}\mathbf{r} + \mathbf{e}, \tag{1}$$

where $\mathbf{p}$ is a $M \times 1$ sensor response vector from the M channel sensors, S is a $M \times N$ matrix of a set of spectral sensitivities, L denotes a $N \times N$ diagonal matrix for recording illuminant. $\mathbf{e}$ is a $M \times 1$ additive noise. For abbreviation, let $S_L = SL$ below. Let denote the projection matrix onto the human visual subspace (HVSS) as P_v. The projected vector $P_v\mathbf{r}$ is termed a fundamental vector below, since the visual system is dependent only on the component of the vector that lies in the HVSS. If $\hat{\mathbf{r}}$ represents a recovered spectral reflectance by the Wiener estimation, the recovered and actual fundamental vector error $\Delta\mathbf{r}_h$ is given by

$$\Delta\mathbf{r}_h = P_v\mathbf{r} - P_v R_{SS} S_L^T (S_L R_{SS} S_L^T + \sigma_e^2 I)^{-1}\mathbf{p} \tag{2}$$

where R_{SS} is the autocorrelation matrix of the spectral reflectance of samples and σ_e^2 is the noise variance and I represents the identity matrix. Since the R_{SS} is symmetrical, it is represented by a set of eigenvectors and eigenvalues of the matrix $R_{SS} = V\Lambda V^T$, where V represents a $N \times N$ basis matrix. Λ is a $N \times N$ diagonal matrix of eigenvalues of the matrix. Let $S_L^V = S_L V \Lambda^{1/2}$. The SVD of the matrix is given by

$$S_L^V = \sum_{i=1}^{\beta} \kappa_i^v \mathbf{d}_i^v \mathbf{b}_i^{vT},$$

where $\beta = \mathrm{Rank}(S_L^V)$, and κ_i^v and $\mathbf{b}_i^v$ represent the i-th singular value, the i-th right singular vector, respectively. By substituting these relations into Eq.(2) and averaging the Euclid-norm of the error vectors $\|\Delta\mathbf{r}_h\|^2$ over the surface reflectances, the mean square error (MSE) is given by[2]

$$E\left\{\|\mathbf{r}_h\|^2\right\} = \sum_{i=1}^{\alpha}\left\|\mathbf{a}_i^v\right\|^2 - \sum_{i=1}^{\alpha}\left\|P_{CV}\mathbf{a}_i^v\right\|^2 + \sum_{i=1}^{\alpha}\sum_{j=1}^{\beta}\left(\frac{\sigma_e^2}{\kappa_j^{v\,2}+\sigma_e^2}\right)(\mathbf{b}_j^{v\,T}\mathbf{a}_i^v)^2, \quad (3)$$

where P_{CV} is the projection matrix onto the subspace spanned by a set of basis vectors

$$\left\{\mathbf{b}_i^v\right\}_{i=1,2,\cdots,\beta}.$$

A column vector $\mathbf{a}_i^v$ is given by

$$\mathbf{a}_i^v = \Lambda^{1/2}V^t\mathbf{a}_i,$$

where $\mathbf{a}_{i,i=1,\cdots,\alpha}$ represent orthogonal vector which span the HVSS. The first and the second terms on the right hand side of Eq.(3) represent the MSE for the noiseless case. Let denote this MSE as MSE_{free}. Therefore the MSE_{free} is given by

$$MSE_{free} = \sum_{i=1}^{\alpha}\left\|\mathbf{a}_i^v\right\|^2 - \sum_{i=1}^{\alpha}\left\|P_{CV}\mathbf{a}_i^v\right\|^2. \quad (4)$$

The third term represents the increase in the MSE in the presence of noise. Therefore we denote this term as the MSE_{noise} and it is represented by

$$MSE_{noise} = \sum_{i=1}^{\alpha}\sum_{j=1}^{\beta}\left(\frac{\sigma_e^2}{\kappa_j^{v\,2}+\sigma_e^2}\right)(\mathbf{b}_j^{v\,T}\mathbf{a}_i^v)^2. \quad (5)$$

If the square of the singular values $\kappa_i^{v\,2}$ is larger than the noise variance σ_e^2, i.e.,

$$\kappa_i^{v\,2}{}_{\;i=1,\cdots,\beta} >> \sigma_e^2,$$

then the MSE_{noise} is negligible. Therefore, if a set of sensors with large values of

$$\kappa_i^{v\,2}$$

is used, the noise effect would be suppressed.

Experimental Results

Spectral sensitivities are formulated as the Gaussian distribution functions, $\exp(-(\lambda-\lambda_i)^2/0.367w_i^{\,2})$, where λ, λ_i and w_i denote a wavelength, a peak wavelength and a bandwidth for the i-th sensor, respectively. Spectral reflectances of a Kodak Q60R1 color chart (228 colors), a Macbeth ColorChecker (24 colors) and a Munsell color chip set (641 colors) were used. Throughout this paper the CIE incandescent illuminant A, CIE daylight illuminant D65 and CIE fluorescent illuminant F2 were used as a recording illuminant and only the CIE daylight illuminant D65 was used as a viewing illuminant. The optimal spectral sensitivities are defined as a set of sensitivities which gives minimum MSE.

Since the intensity of the signals is limited by practical limitations, it is necessary to include a constraint on the signal power. This constraint can be expressed as[1]

$$\rho = \mathrm{Tr}\left(S_L R_{SS} S_L^T\right) \quad (6)$$

where ρ is a constant for each sensor set and the relation of $\rho = 1$ was used in this paper. The noise was assumed to be signal independent and zero-mean (per-channel) with variance σ_e^2. The SNR is defined as[1]

$$SNR = 10\log\left(\frac{\mathrm{Tr}\left(S_L R_{SS} S_L^T\right)}{\sigma_e^2}\right). \quad (7)$$

The noise variance for each SNR was estimated using Eq. (7) and it was used for the Wiener filter. The MSE was computed by averaging the square of the Euclid-norm of fundamental vector differences using Eqs. (1) and (2) for all reflectances.

The typical examples of the optimal sets of spectral sensitivity for the five channels at SNR's of ∞, 50 and 30dB are represented in Figs. 1. The results were obtained for the Macbeth ColorChecker under the D65 for the recording and the viewing illuminant. The results indicate that the optimal bandwidth for each channel is broad and that the peak wavelengths locate to include the subspace that spans the human visual subspace (HVSS) at the noiseless case. However the results show that the spectral sensitivity of each channel tends to sharpen and the peak wavelengths of the sensitivities tend to locate at equal intervals with a decrease in SNR. The same tendencies were also confirmed for the three, four and six sensors under the various combinations of color charts and recording illuminants.

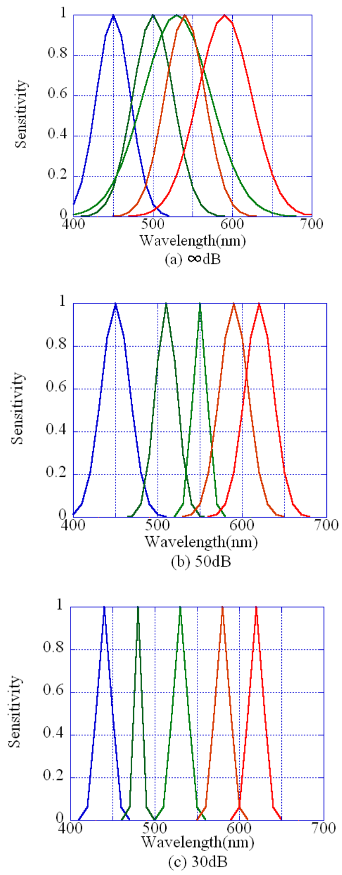

Figure 1. Optimal five channel sensors

To study the influence of noise on the number of the effective sensors, the relationship of the MSE at an optimal set of sensors to the number of the sensors at SNR's of 30, 40, 50, and 60dB are shown in Fig. 2. The results were obtained for a set of three to six sensors that minimize the MSE for a Macbeth ColorChecker under the recording illuminants of the CIE-A. The results indicate that the most significant decreases in the MSE is achieved when the number of sensors increases from three to four and that there is a little improvement over the four at 40 and 30dB. The gradual decreases in the MSE with an increase in the number of sensors are observed at SNR's of 50 and 60dB (which may not be seen in this illustration). It is similar to the results of the optimal nonnegative filters obtained by Sharma et al which was obtained for $\Delta Eab*$ in CIELAB space as a function of the number of channels[4,5]

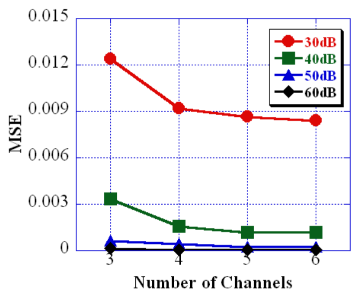

Figure 2. MSE as a function of channel numbers

The six channel sensors optimized at SNR's of 30 and ∞ dB for a Macbeth ColorChecker under the recording illuminant of the D65 were evaluated at various noise levels and the results are represented in Fig. 3. In the figure, the MSE, MSE_{free} and MSE_{noise} are represented as a function of the SNR's. For a set of sensors optimized at 30dB, the values of the MSE_{noise} exceed that of the MSE_{free} at the SNR's below the 50dB and the values of the MSE_{noise} is negligible compared with that of the MSE_{free} at the SNR's above the 50dB. Therefore, the MSE is mainly determined by the MSE_{noise} at the SNR's below the 50dB. On the other hand the sensors optimized at ∞ dB, the values of the MSE_{noise} significantly exceed that of the MSE_{free} at the SNR's below the 60dB. Therefore the values of the MSE are determined by the MSE_{noise} at the SNR's below the 60dB. The results indicate that a set of sensors optimized at ∞ dB is more sensitive to noise compared with that optimized at 30dB.
To study the influence of noise on the optimal number of channels, the MSE, MSE_{noise} and MSE_{free} as a function of channel number at the optimal condition were investigated. Figure 4 shows the results for a Macbeth ColorChecker under the recording illuminant of CIE-A at 30 and 40dB. The figures show that the MSE_{free} decreases with increasing the channel number at 30 and 40dB. The results indicate that an increase in the number of channels is effective to decrease the MSE_{free}. On the other hand the MSE_{noise} increases when channel number increases from three to four as seen in Fig.4(a). At 40dB, a significant improvement in the MSE_{noise} is achieved when the channel number increases from three to four and there is little improvement over the four. The results show that the MSE_{noise} increases with an increase in the channel number at low SNR.

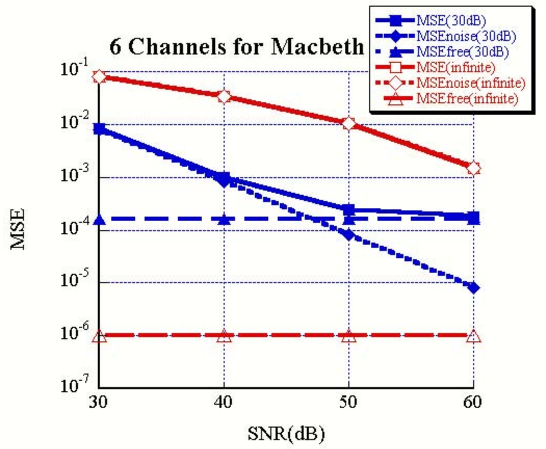

Figure 3. The MSE, MSE_{free} and MSE_{noise} as a function of the SNR's

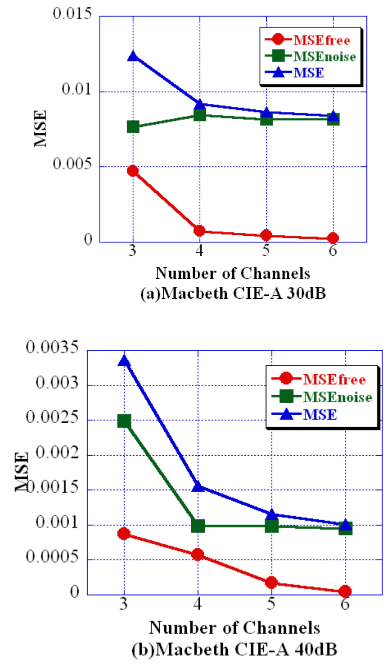

Figure 4. The MSE, MSE_{noise} and MSE_{free} as a function of channel number at the optimal condition

It is interest to consider the reason whey the spectral sensitivities tend to sharpen and the peak wavelengths tend to locate at equal intervals over the visible wavelength with a decrease in SNR and whey a set of sensors optimized at ∞ dB and a set of sensors with the large number of channels is more sensitive to the noise present in a device. To make it clear, the squares of the singular values

$$\kappa_i^{v\,2} ,i=1,2,\cdots,\beta$$

of a matrix S_L^V were computed for all optimal sets of spectral sensitivities for various number of channels at SNR's of ∞, 60, 50, 40, and 30dB.

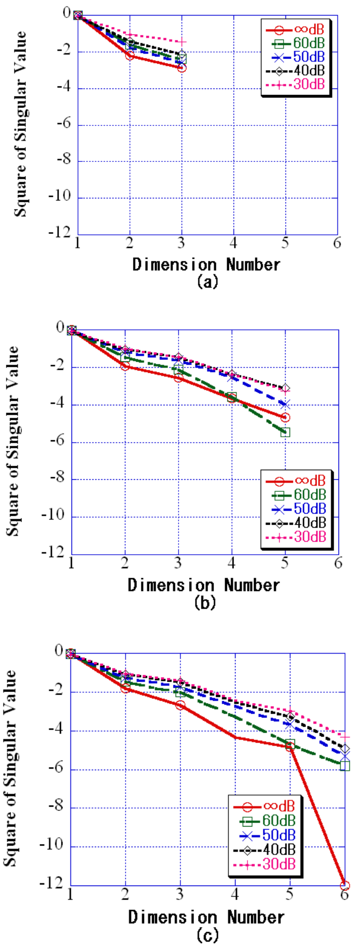

Figure 5. The $\log_{10} \kappa_i^{v^2}{}_{,i=1,2,\cdots,\beta}$ *are plotted as a function of dimension i.*

Typical examples are given in Fig. 5, in which

$$\log_{10} \kappa_i^{v^2}{}_{,i=1,2,\cdots,\beta}$$

are plotted as a function of dimension i. Fig.5 represents the results for the three, five and six channels to capture the image on a Macbeth ColorChecker under the recording illuminant of the CIE-A at SNR's of ∞, 60, 50, 40 and 30dB. From the results, it is confirmed that the values of the square of singular values

$$\kappa_i^{v^2}{}_{,i=1,2,\cdots,\beta}$$

at the $i = \beta$ decrease with an increase in the number of channels at the same SNR and that the values increase with a decrease in the SNR at the same channel number. If the square of the singular values of

$$\kappa_i^{v^2}$$

is larger than the noise variance σ_e^2, i.e.,

$$\kappa_i^{v^2}{}_{i=1-\beta} >> \sigma_e^2,$$

then the third term in the right hand side of eq. (3), i.e., MSE_{noise}, is negligible. Therefore, the spectral sharpening and the location of peak wavelengths at equal intervals in the presence of noise is considered to increase the singular values of

$$\kappa_i^{v^2}$$

to suppress the noise's contribution to errors. The singular values of the matrix at the higher dimension numbers increases if the columns or the rows are linearly independent. Therefore, if the sharp and equi-spaced peak wavelength spectral sensitivities are used then the singular values of a matrix S_L^V would be large at the higher dimensions. From the consideration, the reason for a set of sensors optimized at noiseless condition or a set of sensors with large number of channels is more sensitive to noise is concluded that the MSE_{noise} increases with a decrease in the singular values of the sensors.

The results in the above indicate that the spectral sensitivities with the Gaussian curves must be optimized for a specific SNR and that the estimation of the noise level of a color image acquisition device is essential for the optimization.

Conclusion

A set of single peak spectral sensitivities with the Gaussian distribution function was optimized based on a proposed colorimetric evaluation model to evaluate colorimetric performance of a color image acquisition device. We have shown that the optimization is to increase the singular values of a matrix $SLV\Lambda^{1/2}$ to suppress the noise's contribution to color errors, where S, L, V and Λ represent a sensor matrix, a diagonal matrix for an illuminant, a basis matrix and a diagonal matrix with eigenvalues of an autocorrelation matrix for reflectance spectra, respectively, and that the spectral sensitivities with the Gaussian curves must be optimized for a specific SNR.

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Biography

Noriyuki Shimano recceived the M.S. degree and the Ph.D degree from the University of Osaka Prefecture in 1973 and in 1980, respectively. After the work in Oki Electric Industrial Company, since 1993 he works in Kinki university, where he is associate proffesor in the department of imformatics, school of science and engineerig. His current interests include color research and application, and computer vision.