

Estimation of Just-Noticeable Differences in Multispectral Color Space

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Abstract

This paper presents a method to determine just-noticeable differences in multispectral color space. The basic idea is to find a meaningful connection between (u^*, v^*) chromaticity diagram and multispectral color space. The (u^*, v^*) chromaticity diagram is in focus for the reason that just-noticeable differences, which are Euclidean distances in (u^*, v^*) diagram, have about certain constant value. The visual equivalence of two multispectral vectors is examined based on a metric, which produces values linearly dependent on the Euclidean distance. The paper introduces a metric of that kind. The experiment was carried out on Munsell database measured by the University of Joensuu. The method presented can be used in quality control, image processing, and remote sensing to verify if multispectral colors are constant for the observer.

Introduction

Confusability scaling means a procedure in which it is required to discriminate between color stimuli that evoke only slight differences in visual perception. The stimulus confusions are defined by the just-noticeable difference (jnd), which is sometimes also called just-perceptible difference (jpd). The jnd concept is usually associated with the concept of threshold. The number of jnds is then calculated to generate a scale, which is usually an interval scale.²

Krantz¹² presented a discussion of the invariance of confusability (jnd) scales. A mathematically formal exposition of a theory of discrimination has been given by Falmagne.¹³ Furthermore, the Weber law and the Fechner law play key roles in the basic concepts of confusability scaling.

One of the color analysis challenges is to determine just-noticeable color differences. In 1942 MacAdam ran an experiment to find visual equivalence thresholds. He discovered that standard deviations of colors, which are constant for the observer, are in fact ellipses. Ideally, just-noticeable differences are desired to be identical along any direction in the chromaticity surface, enabling to get just-noticeable difference for any color. There is no uniform color space developed, but some approximations exist. In 1976 CIE introduced $L^*a^*b^*$ and $L^*u^*v^*$ color spaces. In the corresponding chromaticity diagrams some MacAdam ellipses approach circles. MacAdams research was later improved and extended to three dimensions creating the Brown-MacAdam ellipsoids.³ Simon⁴ utilized MacAdams'

results with a modified Munsell-type lightness calculation to provide sets of Simon-Goodwin color-difference charts. They can be used for rapid hand calculation of color differences and also as color-tolerance charts. MacAdam, and Chickering^{6,7} presented two color-difference equations known as FMC-1 and FMC-2. Also Brown,⁸ Wyszecki⁹ measured threshold ellipsoids using similar techniques. Wright¹⁰ measured color discrimination at a number of locations throughout the chromaticity diagram creating suprathreshold loci, not ellipsoids. In fact, this research did not directly result in color-difference equations, but were important in increasing understanding of color-difference perception.¹¹

In 1958 Farnsworth developed a non-linear transformation of the CIE 1931 (x, y) chromaticity diagram, in which deviations from circles are not greater than 20% with respect to radius and in most cases are less than 10%.² In 1944, a complicated model of the about uniform surface was constructed based on properties of MacAdam line element. On the surface, MacAdam ellipses are circles of identical size.

The $L^*u^*v^*$ Color Coordinate System

The method presented in this paper was applied to the (u^*, v^*) chromaticity diagram (Fig. 1). The idea of the method is to project just-noticeable color threshold from the chromaticity diagram to multispectral color space. The projection is possible because of linear dependence between Euclidean distance in (u^*, v^*) diagram and difference between vectors in multispectral color space. The latter is determined through metrics (Eq. (1)-(7)).

The $L^*u^*v^*$ color space is defined as follows:

$$L^* = 116 \left(\frac{Y}{Y_n} \right)^{1/3} - 16$$

$$u^* = 13L^* (u' - u_n')$$

$$v^* = 13L^* (v' - v_n')$$

with the constraint $\frac{Y}{Y_n} > 0.01$. For the values

$\frac{Y}{Y_n} < 0.008856$, the following formula L_m^* is used²:

$$L_m^* = 903.3 \frac{Y}{Y_n}$$

$$u' = \frac{4X}{X + 15Y + 3Z} \quad v' = \frac{4Y}{X + 15Y + 3Z}$$

$$u'_n = \frac{4X_n}{X_n + 15Y_n + 3Z_n} \quad v'_n = \frac{9Y_n}{X_n + 15Y_n + 3Z_n}$$

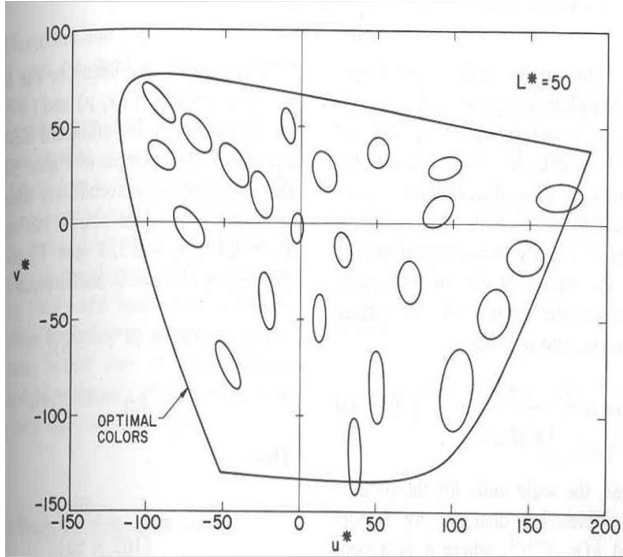


Figure 1. MacAdam ellipses in the (u^*, v^*) chromaticity diagram

The diagram using coordinates (u^*, v^*) is referred to the CIE (u^*, v^*) diagram. The points on the diagram are not uniquely related to chromaticities because their position depends on the value of L^* .² However, for the constant L^* , straight lines in the CIE 31 chromaticity diagram remain straight in the (u^*, v^*) diagram. Euclidean distance on the diagram is calculated as follows:

$$\Delta E = \left((u^*)^2 + (v^*)^2 \right)^{1/2}$$

Experiment with Spectra

In the study, twelve metrics from Ref. [1] and one constructed by our group have been tested. All the metrics take two color vectors to be compared as input and generate a real number from the range from 0 to 1, where "1" means "identical". The metrics are represented as functions $S(x_i, x_j)$, where x_i and x_j are p -bands multispectral color vectors.¹ As a result, the following metrics given linear dependence with Euclidean distance in (u^*, v^*) chromaticity diagram:

$$S = \frac{\cos(\vartheta)(|x_i|^2 + |x_j|^2 - 2|x_i||x_j|\cos(\vartheta))^{1/2}}{\cos(\vartheta)(|x_i|^2 + |x_j|^2 + 2|x_i||x_j|\cos(\vartheta))^{1/2}} \quad (1)$$

$$S = \exp\left(-\beta \sum_{k=1}^p |x_{ik} - x_{jk}|\right) \quad (2)$$

$$S = 1 - \beta \sum_{k=1}^p |x_{ik} - x_{jk}| \quad (3)$$

$$S = \frac{\sum_{k=1}^p \min(x_{ik}, x_{jk})}{\sum_{k=1}^p \max(x_{ik}, x_{jk})} \quad (4)$$

$$S = \frac{\sum_{k=1}^p \min(x_{ik}, x_{jk})}{\frac{1}{2} \sum_{k=1}^p (x_{ik} + x_{jk})} \quad (5)$$

$$S = \frac{\sum_{k=1}^p \min(x_{ik}, x_{jk})}{\sum_{k=1}^p (x_{ik} x_{jk})^{1/2}} \quad (6)$$

$$S = \left(\frac{x_i x_j^t}{|x_i||x_j|} \right) \left(1 - \frac{\|x_i - x_j\|}{\max(|x_i|, |x_j|)} \right), \quad (7)$$

where β is determined through experiment. In our experiment, the value of β was set to 0.00012. The drawback of these metrics is that they produce numbers in a small range. The wider range of values, the easier to distinguish between numbers. The metric from Eq. 7 was constructed by our group on the base of those twelve from Ref. [1]. It was developed on the assumption that the metric comprising angle and distance between vectors will give the best result. In comparing with other metrics, metric from Eq. 7 produces a wider range of numbers for the same range of chromaticity differences (Fig. 2).

Data for the experiment were taken from gloss and matt Munsell databases from University of Joensuu. It represents large amount of color vectors of different chromaticities and luminances. For the experiment, color vectors of different chromaticities and constant luminances were chosen, since we considered Euclidean distance in (u^*, v^*) chromaticity diagram, which has constant luminance. It essentially restricts number of the vectors for testing.

Discussion

A new method to determine just-noticeable differences in multispectral space is introduced. It is based on conversion of just-noticeable differences from $L^*u^*v^*$ to multispectral color space. The difference between vectors in multispectral space is determined using a metric. We found metrics shown linear dependence with Euclidean distance in (u^*, v^*) chromaticity diagram (Fig. 2). Thus, to

compare two multispectral color vectors on visual equivalence is to calculate their difference using metrics (1)- (7) and compare it with certain thresholds under which color vectors are approximately visually equivalent. For the reason $L^*u^*v^*$ is not exactly a uniform color space, the problem to choose the threshold appears. The threshold can be fixed for multispectral vectors producing closely approximated (u^* , v^*) values. In the area of high u^* and high v^* in the (u^* , v^*) chromaticity diagram MacAdam ellipses approaches circles, allowing to set the threshold more accurately. In any case, the method produces approximate results. To improve the method a more uniform color space should be considered.

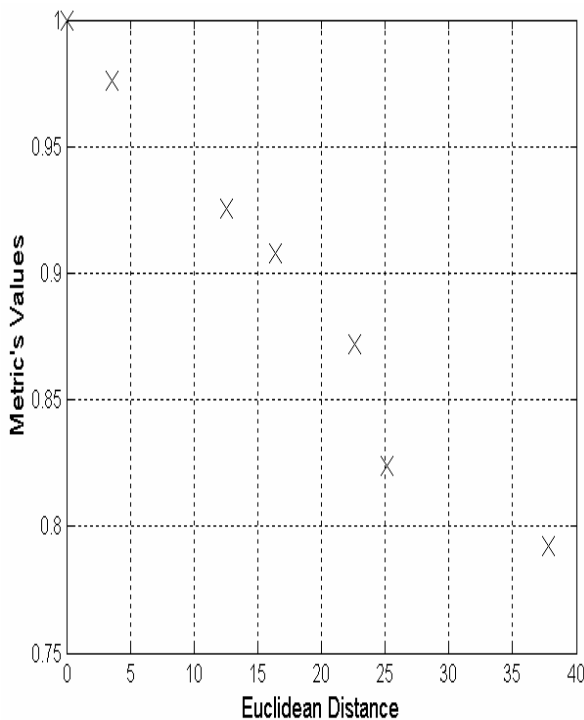


Figure 2. Approximately linear relationship between metric 5's values and Euclidean distance in (u^* , v^*) chromaticity diagram, obtained in testing of Munsell colors matt database.

The study presented in this paper can be used in quality control in paper, textile and ceramic industries. It enables to verify multispectral color to be constant for the observer. However, it can be done for a certain level of luminance, since even MacAdam ellipses are determined for the luminance of 15 milliambert or $48 \text{ cd} \cdot \text{m}^{-2}$. Introduction of luminance is challenge to be studied in x, y chromaticity diagram first. Furthermore, it can be extended to multispectral color space.

References

1. M. Hild, On the effectiveness of color similarity measures in background-frame differencing applications, Osaka Electro-Communication University, Department of Engineering Informatics, Neyagawa, Osaka, Japan.
2. G. Wyszecki, W. S. Styles, Color Science: Concepts and Methods, Quantitative Data and Formulae, Second Edition, John Wiley & Sons, New York, 1982.
3. W. R. J. Brown and D. L. MacAdam, Visual sensitivities to combined chromaticity and luminance differences, Journal of the Optical Society of America, 39, pp. 808-834, 1949.
4. F. T. Simon and W. J. Goodwin, Rapid graphical computation of small color differences, American Dyest. Report, 47 (4), pp. 105-112, 1958.
5. K. D. Chickering, Optimization of the MacAdam-modified 1965 friele color-difference formula, Journal of the Optical Society of America, 57, pp. 537-541, 1967.
6. K. D. Chickering, Color-difference formulas: Clarification concerning usage, Journal of the Optical Society of America, 61, pp. 118-122, 1971.
7. W. R. J. Brown, Color discrimination of twelve observers, Journal of the Optical Society of America, 47, pp. 137-143, 1957.
8. G. Wyszecki and G. H. Fielder, New color-matching ellipses, Journal of the Optical Society of America, 61, pp. 1135-1152, 1971.
10. W. D. Wright, The sensitivity of the eye to small colour differences, Proceedings of the Phys. Society (London), 53, pp. 93-112, 1941.
11. R. S. Berns, Principles of Color Technology, Third Edition, John Wiley & Sons Inc., New York, 2000.

Biographies

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