Adaptive Color Quantization Using the "Baker's Transform"

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Abstract

In this article we propose an original technique to reduce the number of colors contained in an image. This method uses the "Baker's Transform" which allows to obtain a statistically suitable mixture of the pixels of the image. From this mixture, we can extract a square sample, which constitutes a good representation of the initial image. From a sample or from all the possible samples, we can obtain the pallet allowing an adaptive reduction of the number of the colors. We consider four effective methods to obtain this pallet. Finally, we obtain results, which illustrate the good visual quality reached by the reduced images.

Introduction

Everyday we use color images for working or for personal interests. Each one is defined in a color space, RGB very often, with 256 color levels for each channel. So the number of colors available is larger than 16 millions. Most of the time, if we look at the 3D histogram of a color image, we can see that each pixel has a different color level than the others. In fact, we have at the most 2^{16} color levels (2^{24} available) for a 256x256 image. With this constraint, space for storing images increases quickly and the processing time too.

Color reduction is the solution to avoid these problems. To reduce the number of colors, we propose to use the "color quantization" with the aim to replace a color by an other one, which is the closest according with a given criterion.[1,2] This color is chosen in a pallet containing a defined number of colors. Some techniques are available to process color quantization.[3-6] The main goal of these methods is to keep the original appearance of the image for the user.

In this paper we propose a new adaptive method of color quantization based on the use of the "Baker's Transform (BT)".[7,8] In the first part we present this transformation. Then we explain some ways to use this technique to reduce the number of colors of an image. In the third part, we compare the different approaches we propose by using a quality measure of color reduction and we present some results. We conclude with perspectives of this work.

I. The Baker's Transform (BT)

The Baker's Transform (BT for short) is based on the definition of mixing dynamical systems.[9,10] The main interest of these transformations is they mix in a very homogeneous way all the elements of the involved space.

We borrow the following intuitive definition from Ref. [9] p.18: "Let M be a shaker filled with an incompressible liquid comprising 10% of gin and 90% of martini. Let us suppose that the gin initially occupies a portion A of M. After n turns f with a spoon, the gin occupies the portion fn(A). Physically, it is natural to hope that, for n rather large, the proportion of gin contained in an unspecified volume B of M will be about 10%". The interesting point is that every part A of M appears (asymptotically, after mixing) as a "small-scale model" of M.

Arnold & Avez[9] give a lot of examples of such mixing transformations, which are defined on the unit square $[0,1]\times[0,1]$. We have used one of them, the BT. We just mention here that all the examples given by Arnold & Avez are defined on continuous sets. On the other hand, digital images are finite sets of points (pixels). Unfortunately, it appears that a transformation of a finite set is never mixing. But for some peculiar mixing transformations like BT, when restricting to finite sets, it remains some mixing like properties: the pixels are statistically well mixed when iterating the transformation. We call such a transformation a Quasi-Mixing Transformation (QM-T for short).

One iteration of the BT is based on two steps. First, from an original image (see Figure 1), use an "affine" transformation which gives an image twice larger and half high (the total number of pixels remains unchanged) as shown in Figure 2. To do so, we interlace the lines of the image two by two. Then, cut vertically in the middle the resulting image and to put the right half on the left half (the final image has the same size as the original image) as shown in Figure 3.

The BT requires that the image size is 2^Nx2^N pixels and we can show that the BT is periodic with period equal to 4N iterations (any one-to-one transformation of a finite set is periodic). The image is well mixed with N iterations as shown in Figure 4. Now, if we apply 3N additional iterations of the BT to image of Figure 4, we recover the original image. If image size is 2^8 x 2^8 then the well-mixed image is acquired with the 8^{th} iteration and the 32^{nd} iteration gives the original image.

Figure 1. Original 256 x 256 image

Figure 2. First step of initial TB iteration

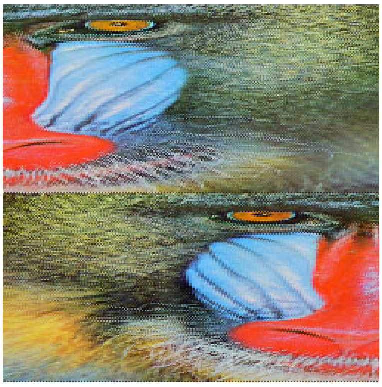


Figure 3. Second step of initial TB iteration

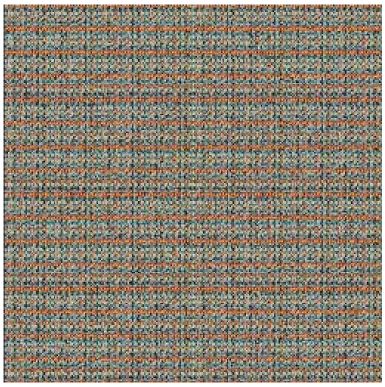


Figure 4. Transformed image at the 8^{th} iteration

The observation of the well-mixed image (see Figure 4) makes it possible to note a kind of double periodicity. Unlike the continuous case for which any part of the "mixed" space has the right percentages of the various components of the space, in the finite case, if such a property is partially preserved, it could be only for certain parts of the mixed image, which it is necessary to choose by taking account of this double periodicity.

Among the properties of QMT, there is one, which is particularly well suited for our purpose, namely the Local Reconstruction Property (see Ref. [8] for the other properties of the BT). For an image of size 2^Nx2^N, let us consider a partition in blocks of size 2^px2^p (p<N). Therefore, from the image of Figure 4 (2^N=256), it is possible to get 4 blocks of 128x128 pixels. For each of these blocks, if we apply the BT (adapted to the size of the image) 3p times (here p=7), we find an image of small size. If we take p=6, we obtain a partition of the image in 16 blocks of 64x64 pixels, and the same can be done for each one of these blocks: if we apply the BT (adapted to the size of the image) 3p times, we find an image as shown in figure 5, and so on. There is thus here a “multi-scale” aspect. Note that if the choice of the block extracted from the well-mixed image is not done according to the rule mentioned above, the reconstruction is not done, in general, in a satisfactory way. This property of local reconstruction illustrates the quasi-mixing nature of the BT.

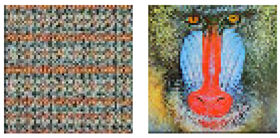


Figure 5. a) 2^6 x 2^6 window from the mixed image and b) result after 18 iterations

II. Color Reduction

As shown above, a little image of size 2^px2^p gives a good representation of the original image for shapes, textures and colors. So the idea is to use one of these windows as a color pallet to reduce all the color levels of the original image. With a 2^Nx2^N image, it is possible to propose pallets containing 2^n colors (n even and n<N). So the number of different pallets available from one image is given by the number $K = 2^{2N-n}$. In this paper we focus our interest on pallets which contains 256, 64, 16 or 4 colors corresponding to extracted windows of size 2^4x2^4, 2^3x2^3, 2^2x2^2, 2^1x2^1 (ie n=8, 6, 4, 2). Given a pallet, the common principle is, for each pixel, to compute the Euclidean distance between its color and all of the colors of the pallet. Then the new color assigned to the pixel is the one that minimizes the distance. The problem is “how to choose the representative window to build the good pallet?”. We propose four solutions.

II.1 Random Choice

A first way consists to randomly choose one of the K available windows with respect to the number of colors wanted. Results from this method and which are presented (in Figure 7 for example) are obtained with the first window available in top and left of the mixed image.

II.2 Creation of a New Pallet by Using the Median of the Colors

The aim is now to create a new pallet from the K windows. For a given position of the pallet, we choose the color as the median of the values in the K available pallets. We process this technique for all the positions of the pallet. Then we have to reduce the colors of the original image by using the algorithm previously described.

II.3 Selection of a Median Pallet

An other possible way is to choose one of the K pallets. To perform this choice, we use a computation of a distance between all the pallets. Then we keep the pallet that gives the median distance. The equation used for this distance, $Sigma_k$, is given by:

$$Sigma_k = \sum_{i=1}^{2^{n/2}} \sum_{j=1}^{2^{n/2}} R_k(i,j) + G_k(i,j) + B_k(i,j) \quad (1)$$

In this equation, “i” and “j” are the color position in the pallet and k is the index of the pallet. R, G, B are respectively the byte of the color Red, Green or Blue. Then we look for the pallet "k" which has the median value $Sigma_k$.

II.4 Creation of a New Pallet by Using the Mean of the Colors

The last solution we propose is, as the second one, to create a $(K+1)^{th}$ pallet from the K initial pallets. But now, the choice of the final color at each position of the pallet is based on the mean of the K values available at this same position from the K pallets.

III. Discussion

For each case, we estimate the quality of color reduction, by computing, for each channel, a distance between the original and the reduced image using the next equation where I_1 and I_2 are the color levels of a pixel respectively from the original ant the reduced image.

$$delta = \frac{\sum_{i=1}^{2^N} \sum_{j=1}^{2^N} \left| I_1(i,j) - I_2(i,j) \right|}{2^N \times 2^N} \quad (2)$$

In Table 1, we present values of “delta” computed for the four methods on the initial image shown at Figure 1. We can see on these values that the third approach, “median pallet”, is always the best when we take the three channels into account. Some time, it is possible to obtain a better result with the first method as we can see for the green channel. But it is a random result and *a priori* we can not know the pallet to use. These results and conclusions are similar on others images.

A first estimate indicates that we need *delta>10* to distinguish the reduction from the original (with the first glance). This value is not precise and will be redefined. Moreover, this value of delta decreases if the image is not very complex and has few colors. Thus, for a 256 colors reduction, differences between original and reduced image are weak. Figure 6 illustrates the good visual color conservation obtained with the third approach. The results obtained with the three others methods are not shown because they are visually very close to the Figures 6b and 6c.

To illustrate differences between the four approaches, we present the 16 color reduced images (see Figure 7) and the 4 color reduced images (see Figure 8). We see some weakness like contrast/brightness down, lack of details, creation of identical color regions, color dynamic reduction, ... Note these weakness appear for important color reductions. Table 2 shows values of delta obtained from other images and confirms our previous remarks. Figure 9 show these images reduced with 256 colors and obtained with the third approach.

Table 1. Value of delta by channel for the images obtained from the four approaches

Image: "MANDRILL" – Size: 256x256 – Initial coding: 2^{24} colors												
Nb of Colors	***Red***				**Green**				**Blue**			
	Approach				**Approach**				**Approach**			
	1	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
256	5.1	5,6	4,9	6,2	4.8	5,6	4,9	6,3	5.1	5,8	4,9	6,8
64	8.7	9,7	8,8	11,4	7.9	8,7	8,0	10,3	8.6	9,3	8,2	11,5
16	14.4	14,9	13,3	22,6	14.0	15,6	11,8	21,8	19.6	21,3	15,8	27,1
4	30.6	31,3	25,3	34,2	25.4	35,9	38,4	34,7	28.7	44,3	31,1	43,6

Figure 6a. Original image (2^{24} colors) - Figure 6b. Reduced image (256 colors) – Figure 6c. Reduced image (64 colors)

Figure 7. 16 color images obtained from the four approaches

Figure 8. 4 color images obtained from the four approaches

Table 2. Value of delta by channel for other images obtained from the four approaches

Image: "FOOD" – Size: 256x256 – Initial coding: 2^{24} colors

	Red				Green				Blue			
Nb of Colors	Approach 1	2	3	4	Approach 1	2	3	4	Approach 1	2	3	4
256	6,9	10,2	6,7	17,4	7,3	11,8	7,2	21,7	7,2	11,9	7,3	19,0
64	13,9	23,0	12,2	34,7	14,8	26,4	13,2	32,2	13,8	28,5	12,7	39,3
16	23,8	27,9	20,5	49,7	25,9	50,5	22,8	47,8	22,4	43,4	24,0	48,3
4	73,6	51,1	35,4	58,2	33,5	48,7	36,0	53,0	50,2	66,2	59,2	63,6

Image: "FLOWERS" – Size: 256x256 – Initial coding: 2^{24} colors

	Red				Green				Blue			
Nb of Colors	Approach 1	2	3	4	Approach 1	2	3	4	Approach 1	2	3	4
256	4,5	4,6	4,6	5,3	5,0	5,9	4,9	7,2	4,7	5,4	4,8	6,3
64	7,7	8,5	8,5	11,2	8,7	9,5	8,5	13,0	8,1	10,6	7,7	12,7
16	16,7	19,6	15,8	27,2	18,0	21,9	18,9	29,9	19,2	18,5	13,8	23,8
4	31,5	62,5	26,3	57,7	37,9	43,0	33,0	43,7	40,3	36,8	25,1	41,5

Image: "GIRL" – Size: 256x256 – Initial coding: 2^{24} colors

	Red				Green				Blue			
Nb of Colors	Approach 1	2	3	4	Approach 1	2	3	4	Approach 1	2	3	4
256	3,5	4,0	3,8	4,5	3,3	3,7	3,5	4,0	3,3	3,6	3,5	4,1
64	6,6	7,2	6,7	8,3	5,9	6,4	5,8	6,2	5,9	6,5	6,4	6,5
16	12,8	15,5	12,8	15,9	10,4	11,8	10,0	11,4	10,4	11,5	10,7	10,7
4	27,5	24,3	20,8	25,9	19,2	22,6	18,3	20,3	17,5	21,2	16,4	18,6

Image: "PEPPERS" – Size: 256x256 – Initial coding: 2^{24} colors

	Red				Green				Blue			
Nb of Colors	Approach 1	2	3	4	Approach 1	2	3	4	Approach 1	2	3	4
256	3,8	3,9	3,9	5,0	3,7	3,7	3,7	4,5	3,7	4,1	3,8	5,6
64	6,6	8,7	6,4	12,6	6,5	6,8	6,3	11,9	6,1	9,6	6,6	11,5
16	10,4	18,6	12,5	20,0	10,5	15,9	10,9	32,4	10,0	18,9	9,3	22,7
4	27,7	42,9	37,9	31,3	25,6	24,8	32,2	55,4	33,6	21,9	24,9	26,4

Image: "LENNA" – Size: 256x256 – Initial coding: 2^{24} colors

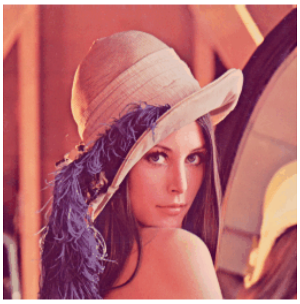
	Red				Green				Blue			
Nb of Colors	Approach 1	2	3	4	Approach 1	2	3	4	Approach 1	2	3	4
256	2,8	2,9	2,6	3,6	2,7	2,9	2,7	3,2	2,8	3,2	2,9	3,3
64	4,8	5,5	4,7	6,2	4,7	5,6	4,9	6,5	4,9	5,1	5,1	7,8
16	7,9	10,9	9,5	19,6	8,1	14,4	7,5	17,2	8,2	12,4	8,0	13,8
4	14,7	25,3	17,8	28,1	25,8	22,6	23,8	27,1	25,2	16,7	18,7	19,5

(a) *(b)*

Figure 9. 256-color images obtained from the third approach a) "Food" and b) "Flowers"

(c) *(d)*

Figure 9. 256-color images obtained from the third approach c) "Girl" and d) "Peppers"



(e)

Figure 11. 256-color images obtained from the third approach e) "Lenna"

Conclusion

In this paper, we present a technique to reduce the colors of an image to a fixed number of levels. This method, based on the use of the "Baker's Transform", is very effective and fully adaptive. This method is also simple because the user specifies only the number of colors that is wanted in the final image. Finally, this method gives good visual results.

We explored four approaches to select the final palette which be used to reduce the color number. The "median pallet" approach gives better results overall. The approach founded on the random selection of the pallet can lead to the same results level, but nothing guarantees the repeatability of these results on various images.

Future works will compare our results with others color quantization algorithm results. We will compare the visual quality and the algorithmic complexity between our method and others. A comparison with usual compression algorithms (as JPEG or GIF) is also envisaged.

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Biographies

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