A Visual Evaluation of the Image Reproduced by Color Decomposition Based on Spectral Approximation for Multiprimary Display

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Abstract

This paper reveals one of the advantages of using multiprimary display, which can utilize the degrees of freedom. A spectral-approximation method, which was already proposed, is one of the methods concerning this advantage. This method is based on minimizing the difference between the spectrum of the original object and the reproduced spectrum under the constraint of tristimulus match for the CIE standard observer. We can expect some reduction of the color mismatch for any CMF. However, visual evaluation for actual images has not been reported until now. Therefore, we made a visual experiment to evaluate the spectral-approximation method using actual images, which contain many colors and shadings. We also present a novel algorithm for the spectral-approximation method.

Introduction

Multispectral imaging and multiprimary displaying are key technologies to realize high-fidelity color image reproduction that is expected to be used in digital archiving, electronic commerce, telemedicine, and so on.¹ One important advantage of using multiprimary display, which has more than three primary colors, is an expansion of the color gamut, and this subject has been actively studied.^{5,6,13} Another advantage of using multiprimary display is to utilize the degrees of freedom for reducing the color mismatch caused by observer metamerism, while conventional trichromatic displays have only one set of display signals that present a given set of XYZ tristimulus values.

The problem of observer metamerism has been recognized.^{8,9} and recently Ohsawa et al. pointed out that the variation of color matching function (CMF) among the observers could not be ignored for accurate color reproduction.² Concerning the advantage of multiprimary display, several methods, which decompose spectrum into primary colors, are proposed to reduce the color mismatch caused by the variation of CMF.^{34,7} Ohsawa et al.³ presented an idea that uses more than three-dimensional CMF derived from a set of CMF data. König et al.⁷ use a

linear programming technique to minimize the color mismatch for a set of CMF data. These methods provide a somewhat optimal solution for a set of CMF data. However, it is difficult to obtain the appropriate observer's CMF data.

On the contrary, no specific CMF data is required in the method proposed by Murakami et al.⁴ This method is based on minimizing the difference between the spectrum of the original object and the reproduced spectrum under constraint of tristimulus match for the CIE standard observer, and we can expect some reduction of the color mismatch for any CMF. They also present the algorithm for this spectral-approximation method and show the effectiveness of the method by simulation and visual evaluations, which uses color patches. However, visual evaluation for actual images has not been reported until now.

Therefore, we made a visual experiment to evaluate the spectral-approximation method using actual images which contain many colors and shadings. In addition, the decomposition algorithm presented by Murakami et al.⁴ is based on a gradient method and requires a significant amount of computation for reproducing image data. In this paper, we present a novel algorithm for the spectralapproximation method, which derives algebraically an optimal solution, and obtains more precise results with less computational cost than Murakami's algorithm.

Decomposition Method Based on Spectral Approximation

In this section, we describe the formulation of the decomposition method based on spectral approximation (spectral-approximation method).⁴

Among the several approaches to decompose spectrum into primary colors,^{3-7,13} the spectral-approximation method is based on the constraint of tristimulus match as the conventional methods.^{5,6,13} We can define the constraint of tristimulus match as follows.

Let N be the number of primary colors, **s** be a target spectrum, n be the number of dimension describing the spectrum, **P** ($n \times N$ matrix) be a primary-spectrum. We present a (scalar value [0, 1] associated with

the linear description of the *i*-th primary by α_i and a set of scalar values by α ($N \times 1$ vector). Assuming that a reproduced spectrum by the display system can be represented by $\mathbf{P}\alpha$, the constraint of tristimulus match can be described by the equation:

$$\mathbf{Xs} = \mathbf{XP}\boldsymbol{\alpha} \tag{1}$$

$$0 \le \alpha_i \le 1, \quad i = 1, \dots, N, \tag{2}$$

where **X** ($3 \times n$ matrix) be a color matching function (CMF). If the number of primary is three, we can get only one solution by

$$\boldsymbol{\alpha} = (\mathbf{X}\mathbf{P})^{-1}\mathbf{X}\mathbf{s},$$
(3)

because **XP** is 3×3 matrix. However, when the number of primaries is greater than three, we have a set of solutions that satisfy eq. (1) because of the degree of freedom.

According to the degree of freedom in multiprimary displays, a selection from a set of solutions that satisfy eq. (1) is required to get a unique set of scalar values α . The conventional methods^{5,6,13} determine the selection rule based on continuity and smoothness of display signals, because the discontinuity may cause spatial noise in the reproduced image. However, concerning the problem of observer metamerism, it becomes appropriate to utilize the degrees of freedom for solving the color mismatch caused by this problem.

Let $\Delta \mathbf{X}$ be a deviation of CMF. We can present a CMF of an observer by $\mathbf{X} + \Delta \mathbf{X}$ and define the color mismatch $E_{XYZ}(\Delta \mathbf{X})$ caused by $\Delta \mathbf{X}$ as follows:

$$E_{XYZ}(\Delta \mathbf{X}) = \| (\mathbf{X} + \Delta \mathbf{X})(\mathbf{s} - \mathbf{P}\boldsymbol{\alpha}) \|$$

= $\| \mathbf{X}(\mathbf{s} - \mathbf{P}\boldsymbol{\alpha}) + \Delta \mathbf{X}(\mathbf{s} - \mathbf{P}\boldsymbol{\alpha}) \|$
= $\| \Delta \mathbf{X}(\mathbf{s} - \mathbf{P}\boldsymbol{\alpha}) \|$
 $\leq \| \Delta \mathbf{X} \| \| \mathbf{s} - \mathbf{P}\boldsymbol{\alpha} \|.$ (4)

If specific information for $\Delta \mathbf{X}$ were available, direct minimization of $\|\Delta \mathbf{X}(\mathbf{s} - \mathbf{P}\boldsymbol{\alpha})\|$ would be a good approach. However, it is usually difficult to obtain the information about $\Delta \mathbf{X}$. At such a case minimization of $\|\mathbf{s} - \mathbf{P}\boldsymbol{\alpha}\|$ becomes a reasonable approach for reducing the color mismatch $E_{XYZ}(\Delta \mathbf{X})$. A spectral-approximation method relies on that idea. Though it is difficult to achieve $\mathbf{s} \cong \mathbf{P}\boldsymbol{\alpha}$ by only few primaries, we can still expect some reduction of the color mismatch for any CMF. Of course, multispectral imaging is necessary to obtain accurate spectrum \mathbf{s} in order to apply this method. We now define an optimal solution of the spectral-approximation method as follows.

Suppose there exist any feasible solutions α that satisfy the constraints (1) and (2), the optimal solution is the solution which minimizes the objective function:

$$E(\boldsymbol{\alpha}) = \left\| \mathbf{s} - \mathbf{P} \boldsymbol{\alpha} \right\|^2$$
⁽⁵⁾

in the set of feasible solutions.

A Novel Algorithm

In this section, we present a novel algorithm for the spectral-approximation method, which derives algebraically an optimal solution, and obtains more precise results with less computational cost than that of the Murakami's algorithm.⁴

A feasible region of the vector $\boldsymbol{\alpha}$ is a convex set, because eq. (1) is a linear constraint and eq. (2) is a convex set.¹⁰ The objective function (eq. (5)) is also convex. Therefore, a local optimal solution becomes a global optimal solution. To get the solution we introduce the Lagrange function:

$$F(\boldsymbol{\alpha},\boldsymbol{\lambda}) = \left\| \mathbf{s} - \mathbf{P} \boldsymbol{\alpha} \right\|^2 + \boldsymbol{\lambda}^t (\mathbf{X} \mathbf{P} \boldsymbol{\alpha} - \mathbf{X} \mathbf{s}), \quad (6)$$

where λ is a vector representation of Lagrange multipliers. Considering the condition:

$$\frac{DF}{D\boldsymbol{\alpha}} = -2\mathbf{P}^{t}\mathbf{s} + 2\mathbf{P}^{t}\mathbf{P}\boldsymbol{\alpha} + (\mathbf{X}\mathbf{P})^{t}\boldsymbol{\lambda} = 0, \qquad (7)$$

and eq. (1), we can write

$$\begin{pmatrix} 2\mathbf{P}^{t}\mathbf{s} \\ \mathbf{X}\mathbf{s} \end{pmatrix} = \begin{pmatrix} 2\mathbf{P}^{t}\mathbf{P} & (\mathbf{X}\mathbf{P})^{t} \\ (\mathbf{X}\mathbf{P}) & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\lambda} \end{pmatrix}$$
(8)

and get the solution $\boldsymbol{\alpha}$ by

$$\begin{pmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} 2\mathbf{P}^{t}\mathbf{P} & (\mathbf{X}\mathbf{P})^{t} \\ (\mathbf{X}\mathbf{P}) & 0 \end{pmatrix}^{-1} \begin{pmatrix} 2\mathbf{P}^{t}\mathbf{s} \\ \mathbf{X}\mathbf{s} \end{pmatrix}.$$
 (9)

If this solution satisfies eq. (2), it is the optimal solution. The algorithm mentioned above is the same as Murakami's algorithm.⁴ When this solution (eq. (9)) does not satisfies eq. (2), Murakami's algorithm finds the optimal solution by a gradient method which requires iterative process to reach the optimal solution. However, a novel algorithm that we present can derive algebraically the optimal solution as follows.

When the solution of eq. (9) does not satisfies eq. (2), the optimal solution exists on a surface of the region determined by eq. (2). Using this fact, the optimal solution can be found in the feasible region where some (1,...,N-3) of the scalar values α_i (i = 1,...,N) are fixed to 0 or 1. The number of possible combinations Cbecomes

$$C =_{N} C_{1} 2^{1} +_{N} C_{2} 2^{2} +, \dots, +_{N} C_{N-3} 2^{N-3}.$$
 (10)

For each combination, we can determine the similar optimization problem for the scalar values α_1 , which elements are not fixed to 1 or 0, as follows:

$$F(\boldsymbol{\alpha}_{1},\boldsymbol{\lambda}_{1}) = \|\mathbf{s} - \mathbf{P}_{0}\boldsymbol{\alpha}_{0} - \mathbf{P}_{1}\boldsymbol{\alpha}_{1}\|^{2} + \boldsymbol{\lambda}_{1}^{t}(\mathbf{X}\mathbf{P}_{1}\boldsymbol{\alpha}_{1} + \mathbf{X}\mathbf{P}_{0}\boldsymbol{\alpha}_{0} - \mathbf{X}\mathbf{s}),$$
(11)

where, $\boldsymbol{\alpha}_0$ are the fixed scalar values (1 or 0), $\boldsymbol{\lambda}_1$ are Lagrange multipliers, and \mathbf{P}_0 , \mathbf{P}_1 are primary-spectrums corresponding to the fixed scalar values and the non-fixed scalar values respectively. Then we can write

$$\begin{pmatrix} \boldsymbol{\alpha}_1 \\ \boldsymbol{\lambda}_1 \end{pmatrix} = \begin{pmatrix} 2\mathbf{P}_1^t \mathbf{P}_1 & (\mathbf{X}\mathbf{P}_1)^t \\ (\mathbf{X}\mathbf{P}_1) & 0 \end{pmatrix}^{-1} \begin{pmatrix} 2\mathbf{P}_1^t (\mathbf{s} - \mathbf{P}_0 \boldsymbol{\alpha}_0) \\ \mathbf{X} (\mathbf{s} - \mathbf{P}_0 \boldsymbol{\alpha}_0) \end{pmatrix}.$$
(12)

The solution $\boldsymbol{\alpha}$ can be made up from $\boldsymbol{\alpha}_0$ and $\boldsymbol{\alpha}_1$. Since the number of combinations C is limited, it takes less computational time than that of the Murakami's algorithm. The optimal solution is the solution which minimize the objective function (eq. (5)) in the feasible region. Additionally, the display system has a bias spectrum in usual. To compensate the bias spectrum **b** in the formulation above we may use $\mathbf{s} - \mathbf{b}$ instead of \mathbf{s} .

Using the algorithm we propose, it becomes easy and practical to apply the spectral-approximation method to image data.

Experiment

We made a visual experiment to evaluate the spectralapproximation method using actual images, which contain many colors and shadings.

The image shown in Fig. 1 is one of the images used in this experiment. We used cloths with pastel colors (light blue and cream yellow) as objects, because these colors have many solutions for the same tristimulus values. The whole light booth was captured to eliminate the visual influence caused by the difference in background. We captured the images using a sixteen-band multispectral camera and displayed them onto a screen using a sixprimary display (DLP $^{\mbox{\tiny TM}}$ projector). The image shown in Fig. 2 is the spectral distribution of the six-primary display. The images on the screen were reproduced with the original size. Since accuracy of color reproduction is important in the experiment, we evaluated the accuracy of the camera and display system.¹¹ The average of ΔE (CIE ΔE_{ab}) in the camera system was 0.92 (Maximum $\Delta E =$ 1.94) about 24 colors of the Macbeth color checker. The average of ΔE in the display system was 1.18 (Maximum $\Delta E = 2.53$) about 210 test colors. Considering these results, we could expect that ΔE in total system is around 2. It is low enough for comparing real objects and reproduced image in the sense of color.



Figure 1. An example image for the visual experiment

The methods compared with the spectralapproximation method were the linear interpolation method,⁶ the matrix switching method,⁵ and a threeprimary displaying method. In the three-primary displaying method we constructed three primaries from the six primaries by restricting the degrees of freedom. As shown in Fig. 3, we set up the light booth beside a screen, and displayed on the screen both the images reproduced by the spectral-approximation method and the image reproduced by one of the other methods. The several observers were asked to select the image that was closer to the real object in the sense of color accuracy including the background (the light booth).



Figure 2. Spectral distribution of a six-primary display

Eleven observers (ten Japanese and one Mexican) that are involved in color reproduction participated in the experiment. The answer "cannot distinguish" was allowed and we removed it from valid answers. The ratio of valid answers is 89.4%. In Table 1 the numbers of selections for each method are compared. For all cases, the hypothesis that the spectral-approximation method is selected with 50% probability can be rejected at the 1% significant level. In this sense, the effectiveness of the spectralapproximation method is suggested. We confirmed that the color differences (ΔE) caused by the decomposition methods were very small (approximately 0.2). The display white is used as the reference white to calculate ΔE .



Figure 3. Layout for the visual experiment

Table 1. Number of selection	ons for each method
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Compared methods	Number of
	selections
Linear interpolation / Spectral-approx.	8/32
Matrix switching / Spectral-approx.	3 / 40
Three-primary / Spectral-approx.	3/32

Discussion

In the experiment, we found variations in answers of the observers. The Mexican observer selected the spectralapproximation method the same number of times as the other methods. On the other hand, four observers always selected the spectral-approximation method. They said that the color differences in one of reproduced images are not negligible, especially for the light booth. It suggests that human is sensitive to the color difference near white color. Comparing the root mean squared error (RMSE) of the light booth's spectrum by simulation, RMSE of the linear interpolation method, the matrix switching method, and the three-primary displaying method were as 1.31, 1.80, 1.12 times large as that of the spectralapproximation method. In Fig. 4, we compare the spectrum of real object and that of reproduced. The spectrum of the spectral-approximation method is closer to the real spectrum than that of other methods.

In the sense of spectral approximation, the spectral distribution of the six-primary display that we used for the experiment cannot be ideal. As Fig. 4 shows, reproduced spectra have many peaks and valleys that may increase the RMSE especially for objects like the light booth, which spectrum is flat and smooth. Additionally RMSE will seriously increase, if we use narrow spectra (e.g. laser lights¹²) to expand a color gamut. Therefore, when designing a spectral distribution of a display, we should concern both a gamut and spectral approximation. The display design is out of scope in this paper. We would like to investigate it in future work.



Figure 4. Comparison between a real spectrum and reproduced spectra

Conclusion

We made a visual experiment to evaluate the spectralapproximation method using actual images which contain many colors and shadings by comparing with other decomposition methods^{5.6} which do not involve spectral approximation. The results suggest the effectiveness of the spectral-approximation method. This method utilizes the advantage of multiprimary display and also relies on the accuracy of the spectral estimation. Therefore, both multispectral imaging and multiprimary displaying are important technologies for high-fidelity color image reproduction.

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References

- M. Yamaguchi, T. Teraji, K. Ohsawa, T. Uchiyama, H. Motomura, Y. Murakami and N. Ohyama, "Color image reproduction based on the multispectral and multiprimary imaging: Experimental evaluation", Proc. SPIE vol. 4663 (Electronic Imaging), pp. 15-26 (2002).
- K. Ohsawa, T. Teraji, F. König, M. Yamaguchi, and N. Ohyama, "Color matching experiment using 6-primary display", Proc. 3rd International Conference on Multispectral Color Science, pp.85-88 (2001).
- K. Ohsawa, F. König, M. Yamaguchi, and N. Ohyama, "Multi-primary display optimized for CIE1931 and CIE1964 color matching functions", AIC Color's 01 2001.
- Y. Murakami, J. Ishii, T. Obi, M. Yamaguchi, and N. Ohyama, "Color conversion method for multi-primary display for spectral color reproduction", to be appeared in Journal of Electronic Imaging.
- T. Ajito, K. Ohsawa, T. Obi, M. Yamaguchi, and N. Ohyama, "Color conversion method for multiprimary display using matrix switching", Optical Review, vol. 8, No. 3, pp. 191-197, (2001).
- H. Motomura, "Color conversion for multi-primary display using linear interpolation on equi-luminance lane method (LIQUID)", Journal of the SID Vol. 11 No. 2 pp. 371-378 (2003).
- F. König, K. Ohsawa, M. Yamaguchi, N. Ohyama, and B. Hill, "A multiprimary display: Discounting observer metamerism", Proc. of SPIE Vol.4421 (9th Congress of ICA), pp. 898-901, (2002).
- 8. Special Metamerism Index: Change in Observer, CIE Publication No.80, Central Bureau of the CIE, Vienna, (1989).
- 9. W. S. Stiles and J. M. Burch, "N.P.L. colour-matching investigation: Final report", Opt. Acta, No.6, pp. 1-26 (1959).
- D. M. Simmons, Nonlinear Programming for operations research, Prentice-Hall, Inc., Englewood Cliffs, N.J., (1995).
- 11. Telecommunications Advancement Organization of Japan, "2002 Annual Report of Natural Vision", (2002).
- 12. http://www.sony.net/SonyInfo/News/Press/200206/02-023E/
- F. König, K. Ohsawa, M. Yamaguchi, N. Ohyama, and B. Hill, "A Multiprimary Display: Optimized Control Values for Displaying Tristimulus Values", Proc. of PICS, Portland, Oregon USA, pp.215-220, 2002.

Biography

Toshio Uchiyama received his M.S. degree in physical electronic engineering from Tokyo Institute of Technology at Tokyo, in 1989. Since 1989 he has worked for NTT Data Corporation. He has been engaged in pattern recognition, image understanding, and multispectral imaging. He was a visiting scholar at the Center for Neural Engineering, University of Southern California, Los Angeles from 1991 to 1993. He has been a researcher of Natural Vision project of TAO Japan from 1999.