# Estimation of Control Values for a 6-Primary Display Considering Different Observers

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## Abstract

The reproduction of colors for the CIE standard observer is definitely determined if display devices with three color channels like a CRT or a TFT display are used. In contrast, multiprimary displays provide several possibilities to reproduce colors for the CIE standard observer. Due to the higher number of variables the degrees of freedom can be used to optimize the color difference between color stimuli of the reproduction and the original for several different observers. The achievement of small reproduction errors for several observers is one of the major advantages, a multiprimary display offers in addition to an essential the enlargement of the color gamut.

In this paper a stochastic algorithm is described to optimize the control of a 6-channel multiprimary display. The aim is to reproduce multispectral input data for any human observer at smallest possible color differences. The origin of the algorithm is the additive mixture of 6 narrow band spectral primaries, which are described by measured spectral power distributions of a laboratory model. The quality of the color reproduction is defined as the maximum color difference between the reference and reproduction for 24 different human observers characterized by their spectral color matching functions. The maximum color difference of the observers calculated in CIE  $\Delta E_{94}$  units is minimized as a function of the 6 control vectors of the primaries. In contrast to previous publications a pseudo gradient procedure is used.

For each specific input spectrum the final control vector of the primaries is achieved by a series of stochastic variations of the control vector. During the modification of the control vector the increment of variation is adapted. This paper demonstrates the effectiveness of the algorithm and shows that the maximum color difference for a the set of 354 representative color stimuli is about  $\Delta E_{94_{max}} \approx 1$  considering e.g.  $D_{50}$ ,  $D_{55}$ ,  $D_{65}$ , or  $D_{75}$  as reference white, and the mean color difference is about  $\Delta E_{94} \approx 0.3$  for the mentioned illuminants.

## Introduction

In recent years multispectral technology has made huge progress through the availability of faster computers to handle the large amount of spectral data. In fact, multispectral image capturing has reached its readiness for marketing. A number of systems have started competing for market share. On the other side neither commercial multispectral displays nor multispectral printers are available so far. Over the past years several groups started working on experimental multiprimary displays [1, 2, 3], but their aims differ. Equipped with up to six primary colors, these displays offer an essential enlargement of the color gamut. A basic problem is the control of more than three primaries because on one hand the colorimetric control is over-determined, but on the other a number of six primaries is not sufficient to realize a good match of spectral color stimuli. This paper contributes to the evaluation of this essential problem.



Figure 1: Experimental setup of a 6-primary display using two LCD-projectors. The spectral power distribution measured in the center of the screen is shown in fig. 2.

#### State of the art

Up to now, laboratory 6-primary displays are realized by using two LCD- or DMD-projectors. The original spectral transmissions of the three channels of the projectors are cut into long wave parts and short wave parts, respectively, by changing the internal optical filters or using additional ones in the optical pathways. As a result, six narrow band channels become available. Each channel is approximatively 50 nm wide (fig. 2). This allows to cover a significantly enlarged color gamut compared to original three channel devices [1, 4, 5] (fig. 3).

#### $\uparrow$ relative intensity



*Figure 2: Spectral power distribution of the laboratory model (measured in the center of the screen).* 



Figure 3: Color gamut of the 6-primary display given in the CIE 1976 u'v'-diagram. The arrows are marking the split of the primaries of the projectors by additional filters. The triangle represents the color gamut of the original projectors.

Another objective of multispectral color technology is not only to realize an enlarged color gamut but also to get rid of any kind of metamerism problems. In contrast to the conventional three primary systems which permit the control for one observer, mainly the CIE 1931 standard observer, a multiprimary system offers additional possibilities. The higher figure of degrees of freedom can be used to minimize observer metamerism. In the case of a self-luminous display, only observer metamerism has to be considered. Nevertheless, some of the published proposals are focusing on the reproduction of a wide gamut for the CIE 1931 standard observer (2°) only [6, 7, 8]. In another proposal six equations are used to match the CIE 1931 standard observer and the CIE 1964 supplementary standard observer (10°) [9] exactly. More advanced proposals are trying to minimize the color difference for a larger number of observers by methods of linear programming or only on the basis of stochastic iteration [10, 11, 12].

In [10], a stochastic iteration process is presented which achieves a maximum color difference of about  $\Delta E_{94} = 1.6$ for 24 observers, the Vrhel data set of representative color stimuli, and assuming illuminant  $D_{65}$ . This result is obtained on the basis of experimental channels but without consideration of the black offset caused by the laboratory setup. If idealized channel characteristics are used the maximum color difference for 24 observers is reduced to about  $\Delta E_{94} = 1.2$ . A method of linear programming [11] yields a mean color difference of about  $\overline{\Delta E_{ab}} \approx 0.5$  by several observers and for the Vrhel data set with illuminant E. In [12], a stochastic iteration is presented using the Vrhel data set, illuminant  $D_{65}$ , and 24 observers. The presented algorithm is able to reduce the maximum color difference for all observers underneath a threshold of  $\Delta E_{94} = 1.5$ . But the algorithm does not really guarantee to find the control vector for the absolute minimum of color differences.

#### Ambition of the algorithm

The stochastic algorithm presented in this paper reduces the color differences of the methods presented in the previous proposals. This was made possible by locating a minimum in the control vector space systematically. A detailed analysis of the control vector space gave hints to reach this aim.

To get comparable results the Vrhel data set of 354 representative color stimuli [13] and 24 observers represented by color matching functions published in [14, 15] are used again (fig. 4).

#### $\uparrow$ tristimulus values



Figure 4: Spectral color matching functions of 24 human observers including the CIE 1931 standard- and the CIE 1964 supplementary standard observers [16, 14], the standard deviator [15] and curves measured by Stiles and Burch [14].

To provide small color differences for every observer the maximum color difference CIE  $\Delta E_{94_{max}}$  is used as a reference. If the mean color difference CIE  $\overline{\Delta E_{94}}$  was used, it

would be smaller but the maximum color difference would rise disproportionately. The use of the  $\Delta E_{94_{max}}$  error criterion guarantees that the optimization will result in color differences between any original color and reproduction which are not noticeable if the final result of the  $\Delta E_{94_{max}}$  criterion is small enough.

In addition to previous publications various light sources are used to verify the performance of the algorithm, including measured and standardized ones [16, 17], e.g.  $D_{50}$ ,  $D_{55}$ ,  $D_{65}$ ,  $D_{75}$ , A, B, C, E, and  $F_1$  to  $F_{12}$ . This set contains illuminants with uniform spectral radiating power on the one hand and spiky spectral power distributions on the other.

## Algorithm

The stochastic algorithm can be divided into different parts. First of all an appropriate starting vector has to be found. Then small variations of the six dimensional control vectors are stochastically generated and reduced maximum color errors are searched for. Yet, if improvements are realized, it is not clear whether the absolute minimum has really been found since there could be many local minima beside the absolute one. The results after this step are comparable to the results of the algorithm presented in [12].

To improve the results, another step of optimization is appended. Further knowledge about the constitution of the six dimensional vector space is taken into account. Since a six dimensional space cannot be displayed, this knowledge is derived from various cross sections of three dimensional subspaces surrounding the results derived in the second step.

The study shows that the six dimensional vector space of maximum color differences shows smooth gradients without greater steps or an irregular spread of minima. This knowledge makes it possible to improve the performance using a gradient based final optimization procedure.

#### **Preliminary remarks**

All calculations are performed in the spectral range from 380 to 720 nm with steps of 1 nm in order to consider even spiky illuminants. The following calculations are based on the spectral characteristics of the experimental system measured in the center of the projection screen. The black offset of the LCD-projectors cannot be neglected. To get proper values, one has to subtract the black offset from the spectral primary characteristics:  $C_n = P_n - K_{offset}$ , where  $C_n$  are the spectral channel characteristics,  $P_n$  are the measured primary characteristics, and  $K_{offset} = \{k_{380nm}, k_{381nm}, \dots, k_{720nm}\}^T$  is the black offset. For calculating the spectral reproduction stimulus R the black offset has to be taken into account once again. The reproduction is  $R = \sum_{n=1}^{N} c_n \cdot C_n + K_{offset}$ , where  $c = \{c_1, c_2, \dots, c_N\}^T$  is the control vector of the display channels and N = 6.

#### Estimation of an adequate starting vector

In order to save time for the main optimization a rapid method to estimate the starting vector is required. Nevertheless it is necessary to get a well chosen starting vector because the quality of the results depends on the choice of the starting vector. An initial point of this algorithm is the assumption that a spectral approximation is a good estimation for starting the following optimization process. A spectral approximation can be found quickly and in consideration of six narrow band channels this is a good vector to start with which has been proven by the results of this procedure.

The estimation can be separated into three steps. In the first step the calculation is started by using the zero vector  $c = \{0, 0, ..., 0\}^T$ . To accelerate the calculation all channels are raised together to approach a sufficient amplitude.

During the first two steps of the estimation the spectral squared error  $e_{spec}^2 = \sum_{n=380nm}^{720nm} (r_n - o_n)^2$  is used as error criterion, where  $r_n$  is a spectral sample of the reproduction stimulus  $R = \{r_{380nm}, r_{381nm}, \dots, r_{720nm}\}^T$  and  $o_n$  is a sample of the original stimulus. If CIE  $\Delta E_{94_{max}}$  is used instead the influence of the channels at the border of the human visibility is weak due to the weighting of the spectra by the color matching functions. Hence, the border channels are not sufficiently taken into account for finding an adequate starting vector.

During the second step the amplitudes of the channels are raised and dropped sequential only, one by the time. This procedure is repeated several times, followed by the same procedure only using  $\Delta E_{94_{max}}$  as error criterion. This additional expense is necessary because it would not be possible to achieve equivalent final results at the end of all parts of the optimization if the starting vector is estimated by using  $\Delta E_{94_{max}}$  only. Regardless of the effort the maximum color difference at the end of this part cannot be reduced to less than  $\Delta E_{94_{max}} = 8$  or  $\Delta E_{94_{max}} = 10$  in the best case for all 24 observers and all spectral stimuli. Hence, the spectral match on its own or an adapted version using the  $\Delta E_{94_{max}}$  criterion is not a suitable estimation of the control vector *c*.

# Preparation of the control vector for the final optimization

During this part of optimization it is intended to decrease the color difference  $\Delta E_{94_{max}}$  on the one hand and to reach a region near a minimum of color differences  $\Delta E_{94_{max}}$  without greater steps or irregular spread of minima of the error criterion on the other hand. This is necessary before starting the final optimization described in the next subsection. So, the intention is to find the magnitude of the minimum near the global minimum.

In order to qualify the optimization's progress the maximum color difference  $\Delta E_{94_{max}}$  for all 24 observers is used as error criterion from now on.

The goal to find a better control vector is solved by the variation of the amplitudes of the channels. A part of the



Figure 5: Simplified principle of the algorithm (without attention to the black offset, the change of the error criterion, and regards to parallel-serial processing)

modulation amplitude of one channel is replaced by variations of the modulation amplitudes of the other channels to substitute its influence on the integral value. Always under consideration of the error criterion, the best fit of the original stimulus is chosen. In different passes the choice of the channel to change is done cyclic or by pseudo random to avoid dead lock between the channels caused by an exclusively cyclic rotation. This preparation already leads to pretty good results and they are comparable to the results performed by [12].

#### **Final optimization**

The final optimization is a gradient based procedure. Due to the fact that the six dimensional vector space is not known in an analytical representation the calculation of the gradient has to be substituted by a pseudo gradient. In the beginning the control vector of the previous optimization is used as reference point. In each dimension three points are used. In case of a six dimensional vector space  $3^6 = 729$  color differences have to be calculated per iteration. The points surround the reference point in a special grid. The point of the smallest color difference is chosen as new reference point. Between each iteration the grid is adapted automatically. The magnitude of the new grid depends on the slope and the condition of the surrounding vector space. To specify the new grid all calculated points of one iteration are taken into account. In this way it is possible to substitute the missing analytical representation of the vector space. The adaption of the grid has to be limited. Otherwise the reference point may stay the same and the grid may toggle between two magnitudes without any progress of minimization of the color difference. If the grid is adapted properly this procedure converges in a few iterations to adequate results. The order the magnitude of the grid has to be adapted

depends on the way the characteristics of the channels were scaled and pre-calculated, of course.

#### $\uparrow$ relative intensity



Figure 6: Spectral power distribution of six ideal primary channels with smooth spectral aperture functions.

### Results

The simulation is carried out on the basis of a real six primary display (fig. 2) and by using ideal channel characteristics (fig. 6). The spectral power distribution is measured at the laboratory model in the center of the screen. For the experimental system the black offset of the display is taken into account. To test the performance of the algorithm, 354 representative color stimuli of the data set of Vrhel [13] are used. The color matching functions of 24 observers are selected from published color matching functions. The color difference  $\Delta E_{94}$  is calculated for 52 different illuminants. The investigations indicate that the color difference of the Table 1: Sample of color differences  $\Delta E_{94}$  between calculated reproduction and original (maximum  $\Delta E_{94max}^{exp.ch.}$ , mean of the 354 maxima of 24 observers ( $\overline{\Delta E_{94max}^{exp.ch.}}$ ), and medium color differences  $\overline{\Delta E_{94}^{exp.ch.}}$  for experimental channels of the laboratory setup) for 354 spectral stimuli at selected light sources.

	$\Delta E_{94_{max}}^{exp.ch.}$	$\overline{\Delta E_{94_{max}}^{exp.ch.}}^{(354)}$	$\Delta E_{94}^{exp.ch.}$
$D_{40}$	1.38	0.49	0.29
$D_{50}$	1.05	0.49	0.28
$D_{55}$	1.06	0.49	0.28
$D_{60}$	1.11	0.50	0.28
$D_{65}$	1.16	0.50	0.29
D <sub>75</sub>	1.24	0.51	0.29
$D_{90}$	1.50	0.53	0.31
Α	1.49	0.61	0.36
В	0.97	0.46	0.26
С	1.05	0.45	0.25
Ε	1.71	0.59	0.35
$F_1$	0.77	0.30	0.18
$F_2$	0.79	0.31	0.19
$F_3$	1.02	0.37	0.23
$F_4$	1.47	0.44	0.28
$F_5$	0.75	0.30	0.19
$F_6$	0.88	0.34	0.20
$F_7$	0.81	0.31	0.19
$F_8$	0.99	0.34	0.21
$F_9$	0.83	0.33	0.20

Table 2: Sample of color differences  $\Delta E_{94}$  between calculated reproduction and original (maximum  $\Delta E_{94max}^{ideal\,ch}$ , mean of the 354 maxima of the 24 observers ( $\overline{\Delta E_{94max}^{ideal\,ch}}$ ,), and medium color differences  $\overline{\Delta E_{94}^{ideal\,ch}}$  for ideal channels) for 354 spectral stimuli at selected light sources.

	$\Delta E_{94max}^{idealch.}$	$\Delta E_{94max}^{idealch.}$ <sup>(354)</sup>	$\Delta E_{94}^{idealch.}$
$D_{40}$	0.63	0.16	0.09
$D_{50}$	0.59	0.18	0.11
$D_{55}$	0.61	0.20	0.13
$D_{60}$	0.72	0.23	0.14
$D_{65}$	0.84	0.26	0.16
$D_{75}$	1.04	0.31	0.19
$D_{90}$	1.27	0.37	0.23
Α	0.54	0.18	0.11
В	0.58	0.17	0.10
С	0.70	0.18	0.12
Ε	1.58	0.47	0.30
$F_1$	0.77	0.22	0.13
$F_2$	0.71	0.24	0.14
$F_3$	0.78	0.28	0.17
$F_4$	0.90	0.33	0.20
$F_5$	0.76	0.21	0.13
$F_6$	0.75	0.25	0.15
$F_7$	0.72	0.21	0.12
$F_8$	0.84	0.22	0.13
$F_9$	0.83	0.23	0.14

experimental six primary display is close to that of a system represented by six ideal channels.

Parts of the results of the study are presented in table 1 for the channels of the experimental setup (fig. 2) and in table 2 for ideal channels (fig. 6) measured in CIE  $\Delta E_{94}$ . The algorithm results in small color differences, e.g. the maximum color difference for  $D_{65}$  of  $\Delta E_{94_{max}} = 1.16$  and  $\overline{\Delta E_{94}} = 0.29$ for the experimental setup (see fig. 7). Nevertheless, even this algorithm does not really guarantee to find the global minimum of all color differences. Yet, it achieves a control vector which provides adequate color differences and we suppose that the achieved results are near the global minimum of color differences.



Figure 7: Color differences in CIE  $\Delta E_{94}$  between original color stimuli of the Vrhel data set and simulated reproduction using the spectral power distribution of the laboratory model (see fig. 2), calculated for 24 observers (see fig. 4) with illuminant D<sub>65</sub> (ordered by size).

Further investigations show that the more the spectral illuminants differ from the ones used in the LCD-projectors the greater the color difference  $\Delta E_{94_{max}}$  between original and reproduction became. The same effect was observed, the farther the illuminants differed from standardized illuminants e.g.  $D_{50}$ ,  $D_{55}$ ,  $D_{65}$ , or  $D_{75}$ , especially if very high or low color temperatures were considered. The color differences are a little bit smaller if ideal channels with smooth spectral aperture functions are used for the reproduction except for illuminants simular to the light source of the projectors.

All results of the experimental six primary system are calculated without consideration of errors caused by quantization and spatial channel irregularities.

If the kind of reference illuminant is similar to the one of the projectors the maximum color difference is near or below  $\Delta E_{94_{max}} = 1$  in the worst case for 354 representative spectral stimuli and for 24 observers described by color matching functions with realistic variations. This is the case for many well-established technical illuminants, which means the spectrum is neither too steep nor too spiky.

# **Conclusions and Outlook**

The algorithm described in this paper provides color reproduction of high fidelity. During the optimization the control vector of the display channels is undergoing several steps. The algorithm achieves good results for the reproduction of spectral stimuli by using the radiation characteristics of an experimental six primary display on the one hand and for ideal channels on the other.

Furthermore, it is demonstrated that six primaries are a good compromise between expense and quality of color reproduction even for many observers with greater variation of color matching functions.

Although, the calculations are carried out using C++ this implementation of the algorithm is still too time consuming for real time processing of images. For future application it is necessary to fasten the calculations. Fortunately the algorithm is well suited for parallel processing because each pixel can be calculated independently.

### References

- [1] T. Ajito, T. Obi, M. Yamaguchi, N. Ohyama, "Expanded color gamut reproduced by new six primary projection display", Proc. Electronic Imaging 2000, Photonics West, SPIE Vol. 3954, San José, CA, USA, 2000
- [2] T. Ajito, T. Obi, M. Yamaguchi, N. Ohyama, "Six-Primary Color Projection Display for Expanded Color Gamut Reproduction", Proc. Int. Symp. on Multispectral Imaging, Chiba University, Japan, 1999, pp. 135-138
- [3] B. Hill, "Aspects of total multispectral image reproduction systems" (invited), Proc. 2nd Intern. Symposium on High Accurate Color Reproduction and Multispectral Imaging, Chiba University, Japan, Oct. 2000, pp. 67-78
- [4] B. Hill, "Optimization of multispectral imaging systems: best spectral match versus least observer metamerism" (invited), Proc. 9th Congress of the International Colour Association (AIC Color 01 Rochester), Rochester, NY, USA, June 2001
- [5] B. Hill, "The History of Multispectral Imaging at Aachen University of Technology", online publ.: http://www.multispectral.org, Spectral News No. 2 Rochester, NY, USA, 2002
- [6] T. Ajito et al., "Color conversion method for multiprimary display using matrix switching", Optical Review, Vol. 8, 2002, pp. 191-197
- [7] F. König, N. Ohyama, B. Hill, K. Ohsawa, M. Yamaguchi, "A Multiprimary Display: Optimized Control Values for Displaying Tristimulus Values", Image Processing, Image Quality and Image Systems Conference PICS, Portland, Oregan, USA, April 7-10, 2002
- [8] H. Motomura, H. Haneishi, M. Yamaguchi, N. Ohyama, "Backward Model for Multi-Primary Display Using Linear Interpolation on Equi-luminance Plane", Proc. IS&T's 10th Color Imaging Conference, Nov. 2002, Scottsdale, AZ, USA, pp. 267-271

- [9] K. Ohsawa, F. König, M. Yamaguchi, N. Ohyama, "Multiprimary display optimized for CIE1931 and CIE1964 color matching functions", Proc. 9th Congress of the International Colour Association (AIC Color 01), Rochester, NY, USA, June 2001, Proc. SPIE Vol. 4421, 2002, pp. 939-942
- [10] B. Hill, "Optimization of multispectral imaging systems: Best spectral match versus observer metamerism", 9th Congress of the Int. Colour Association (AIC Color 01), Rochester, NY, USA, June 2001, Proc. SPIE Vol. 4421, 2002, pp. 481-486
- [11] F. König, K. Ohsawa, M. Yamaguchi, N. Ohyama, B. Hill, "A multiprimary display: Discounting observer metamerism", Proc. 9th Congress of the International Colour Association (AIC Color 01), Rochester, NY, USA, June 2001, Proc. SPIE Vol. 4421, 2002, pp. 898-901
- [12] Th. Boosmann, B. Hill, "Control Algorithm for 6-Primary Displays Minimizing Effects of Observer Metamerism", PICS: The Digital Photography Conference, Rochester, NY, USA, May, 2003
- [13] M.J. Vrhel, R. Gershon, L.S. Iwan, "Measurement and Analysis of Object Reflectance Spectra", Color Res. Appl. 19, No. 1, 1994, pp. 4-9
- [14] G. Wyszecki, W.S. Stiles, "Color Science Concepts and Methods, Quantitative Data and Formulae", Second Edition, John Wiley & Sons, New York, 2000, ISBN 0-471-39918-3
- [15] CIE Tech. Report Publ. No. 80, "Special Metamerism Index: Observer Metamerism", 1989
- [16] CIE Publication 15.2, "Colorimetry, Second Edition", Commission International de l'Eclairage, Vienna, 1986, ISBN 3 900 734 003
- [17] CIE-Division 8: Image Technology, TC 8-04: Adaption under Mixed Illumination Conditions, http://www.colour.org/tc8-04/Data/F.txt

## **Biography**

Thomas Boosmann received his diploma degree in Electrical Engineering from the Aachen University of Technology in 2000. He is now engaged in research on multispectral imaging systems with focus on multichannel displays. He is member of the German society for color science and application DfwG.

Bernhard Hill received his diploma and Dr.-degree in Electrical Engineering from the Aachen University of Technology. In 1968, he joined the Philips Research Laboratory Hamburg, Germany and started research in the field of laser beam detection, laser recording and holography. He became head of the optics group in 1974 and developed erasable magnetooptic memory technology and optical printing devices. In 1984, he changed to the Aachen University of Technology. He is now focused on color management, gamut mapping and multispectral imaging. From 1990 to 1994, he was manager and dean of the faculty. He is member of IS&T, SID, VDE and vice president of the German society for color science and application DfwG, and he is the German representative in CIE - Division 8.