Calculating a Color Gamut Border Using a Grid Approach

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Abstract

Colors are a way of communication that emphasizes the information of a given illustration. Therefore color faithfulness is evident. To know if a given color is within a special color gamut first of all this gamut needs to be known.

To calculate the color gamut of a device a number of solutions have been suggested. Most of them are dealing with the fact that they over- or underestimate the real volume of the color gamut, respectively.

Our approach applies a grid onto the hull of the color volume. Using this approach a very accurate information about the border is received independently of a convex or non-convex shape. Having got this basic grid information it comes out that a cut at a given L* value can be calculated. Therefore for a given color value a slice of the color gamut can be calculated and the relative position can be obtained.

Using this procedure it is also possible to determine the volume of the color gamut.

Introduction

The existence of color gamuts is evident to many users who deal with color reproduction. They have to cope with a number of different color gamuts. E.g. an image capturing device (ICD) usually deals with red, green and blue (RGB) color separation. Also the display unit works with a RGB scheme. Both devices have a special amount of colors that are recognizable or displayable, respectively. These colors are characterized by being described by a color gamut.

An other task is the recreation of e.g. a picture that is present in a computer with given RGB values by printing it onto a piece of paper. This may be a home printer or an offset printing machine. Both kinds of devices use the colors cyan, magenta and yellow (CMY) to create all the wanted colors. Most of the devices apply also a black color additional to CMY (CMYK). The printing devices therefore also have a number of colors createable characterized by a corresponding color gamut. Every device has its own color gamut and therefore a transformation has to be done when passing color values from one device to an other.

There are a number of approaches to deal with the overall process to assure a true color reproduction. E.g. an original is scanned by an ICD, handled inside a computer and printed onto a piece of paper. The most important

proceedings to handle this task is founded by the International Color Consortium (ICC).[1] In this approach a device independent color system (CIE L*a*b*) (Commission Internationale de L'Eclairage) is used to exchange color information. For every device used to capture or display the color information, respectively, a transformation between the device gamut and CIE L*a*b* values is defined. Basic information for this procedure is the color gamut of the corresponding device.

To determine the color gamut of an offset printing machine the simple plot shown in Figure 1 is used sometimes. There a two dimensional mapping of the CIE L*a*b* color system is drawn. Colors with a*b* values placed inside the closed line are claimed to be printable. All other color values are not colorfast printable. In this proceeding the three dimensional shape of the device's color gamut is simply neglected.
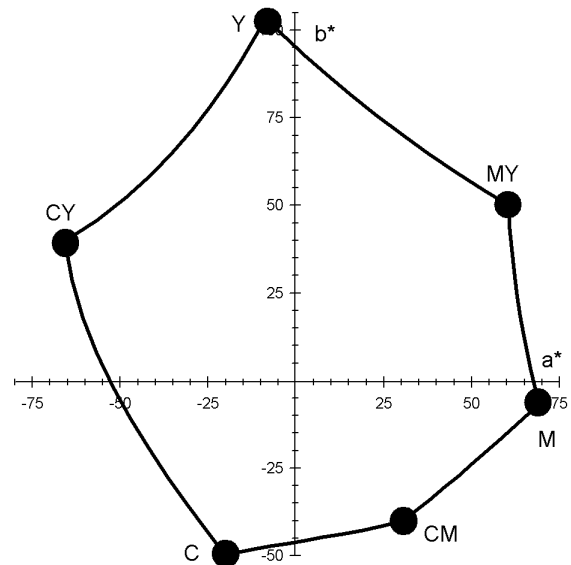

*Figure 1. Primary and secondary colors of CMY(K) in an a*b* plot diagram.*

There are a number of approaches known that deal with the determination of the color gamut of a device. There are even approaches published to calculate the color gamut used by a picture.[2]

The nature of the hull of a color gamut is that it is partly convex but also partly concave. This have to be in mind looking at different approaches in determination of

the color gamut volume. There are indeed some approaches that simply ignore all concave parts of the hull.[3]

In general there are two ways to determine the color gamut of a device. The first one is to work with some color values directly.

One way in ignoring concave parts is the QuickHull (QHull) algorithm.[3,4] It is a robust and fast calculation but overestimates the volume of the color gamut.

Taking this problem into account a number of new solutions have been suggested.

The approach "alpha-shapes" inflates the color gamut volume in such a manner that only a convex hull is present. Then algorithms of the convex hull can be applied and the result is deflated. This algorithm depends on an empirical factor α that can cause errors in the calculation by adding or loosing hull points.[5]

An other approach is to employ ray-tracing with "local" convex hull parts. The total device gamut is subdivided into convex parts and then analyzed using a ray-tracing technology to find the extreme color gamut values.[6]

Segment Maxima Method is a calculation method of a gamut boundary descriptor (SMGBD).[2] Here a number of segments are applied to the CIE L*a*b* coordinated system. In every segment the most extreme value is determined.

The second way to describe the color gamut is to find an analytical color gamut representation. This mathematical model is based on real color values to initialize it. There are a number of papers finding and handling such an algorithm.[7,8]

The need to determine a device's color gamut is much wider than in the printing industry itself. E.g. for some varnish tables there has been investigations in calculation of the maximal extent of the achievable color gamut.[9]

Our approach is to determine the hull points and use their values directly. It is an easy way to determine e.g. a printer's color gamut for some studies. On the other hand it's not sufficient to use it as a general tool like many of the previous mentioned methods are.

Calculation of the Hull

The color gamut of a device is surrounded by a hull. When calculating this hull also the volume of the color gamut is determined. Therefore first of all the hull points are identified. Having calculated these values slices of the color gamut volume can be cut. A number of them will be presented in Figures 4 and 5. Based on these calculations a way to calculate the gamut's volume will be introduced.

First of all our approach is introduced using a CMY color gamut. Then it is adapted to the RGB model. In the end the CMY color gamut is extended to CMYK by adding black into the considerations. All of the following investigations depend on CIE L*a*b* color values that can either be displayed and measured or calculated using an adequate theoretical model.

The CMY Color Gamut

The color gamut of CMY contains all colors that can be achieved mixing cyan, magenta and yellow in any amount from 0% to 100% each. To get a limited number

of colors to work with each color is change with a step width of 10% from 0% to 100%. So each color has 11 different levels of intensity. Working with these three colors the step width of 10% leads to a total of $11^3 = 1,331$ colors created. But not all of these colors are placed on the hull of the color gamut. Therefore first of all the color values building the hull have to be determined out of the whole number.

Using cyan, magenta and yellow as primary colors will yield to red, green and blue as secondary colors. There are an other two additional colors that have to be taken into account: the paper color, usually white, and all three colors printed on top of each other, usually some kind of black.

In Figure 2a the relationship between all the eight colors is depicted. All eight colors are adjusted to the vertices of a cube. The straight lines are possible ways when changing one color at a time. The lines hidden from the cube are broken. All lines have 11 color steps when the starting and ending vertex is included.
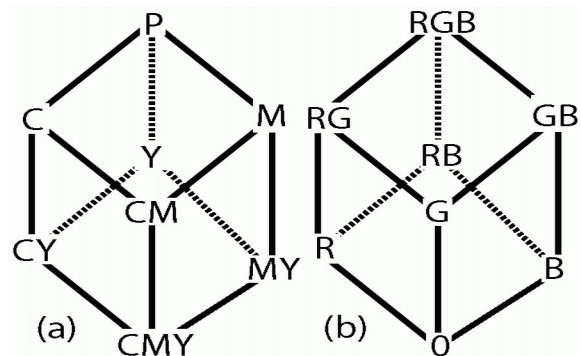

Figure 2. Schematic representation of primary and secondary colors of a (a) CMY device, P: paperwhite, (b) RGB device, 0: no color lit.

One additional information is drawn in Figure 2a implicitly. The highest L* value is that of paper white, the lowest that of CMY. By adding more color the L* value is decreased. But the absolute L* values of cyan, magenta and yellow are different. Therefore the L* axis is not drawn in Figure 2a.

Looking at Figure 2a it is pretty clear that all eight vertices are vertices of the gamut hull. Also the drawn straight lines are edges of the color gamut. In conclusion all $(11-2)^2 = 81$ points inside of every of the six rhombi created by four straight lines each have to be the hull of the color gamut. Adding all these points (8 corners and 12×9 of edge lines and 6×81 of rhombus content) a total number of 602 points are describing the hull of the color gamut. Compared to the total of 1,331 points available it's only 45%. All other 729 points are inside of the color gamut.

At this stage all points are known that are building up the hull of the color gamut. So the first and important step in calculating a cut of the color gamut at a special L* value is done. To do this further calculation it is important to organize the points determined in the previous section in a logical order.
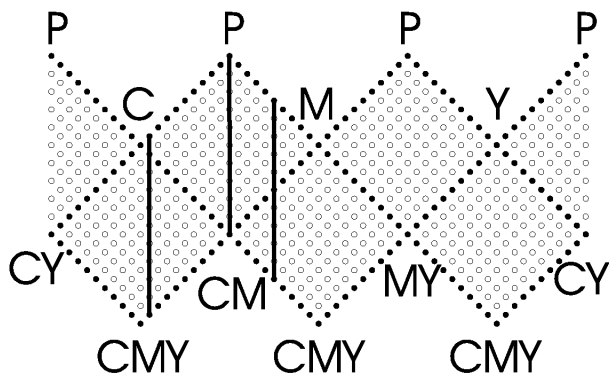
Figure 3. Two dimensional transformation of Figure 2a.

Figure 3 is generated from Figure 2a by organizing the three dimensional plot into a plain diagram. The bold circles are indicating the straight lines from Figure 2a. Some of these lines (from P and to CMY, respectively) are shown multiple because of the plain order. In Figure 3 all grid points are shown explicitly.

The logical order chosen for the hull points is a matrix of the dimension 72×11. In this matrix 72 lines are stored each consisting of 11 points. Each line is monotone according to the L* values. There are 60 vertical lines according to the three example lines given in Figure 3 stored into the matrix. The start and end point of these lines are in the filled circles. Further more all 12 lines indicated by filled circles are stored in the matrix. This makes a total of 792 matrix elements compared to 602 different points.

The calculation of a cut for a given L* value comes out to be pretty simple having done this preparatory work. All 72 lines have to be checked if the given L* value is contained. If so the belonging a* and b* values can be determined using linear regression or any other sufficient procedure.

Having done this procedure for an offset printing process the result is given in Figure 4. Cuts at L* values of 20, 40, 60 and 80 are shown. Additionally the border of the maximal extent of the color gamut is shown. It can be seen that the cuts are fitting inside the border given. Also the changing shape of the cuts can be seen.

When a color location should be checked if it is inside the color gamut than a cut have to be done for that given L* value. Without this proceeding it's nearly impossible to assure that the color location is inside the color gamut.

The RGB Color Gamut

Image processing prior to printing is usually done at a CRT. The CRT is based on using RGB colors. Therefore it's a good supplement to determine the RGB color gamut along with the CMY's one.

The similarity between both systems is the use of three primary colors. The main difference between both systems is that unlike the CMY color gamut calculation that is based on subtractive color mixing the RGB colors are mixed additively. So the procedure developed for CMY can be adapted to RGB in the following way.
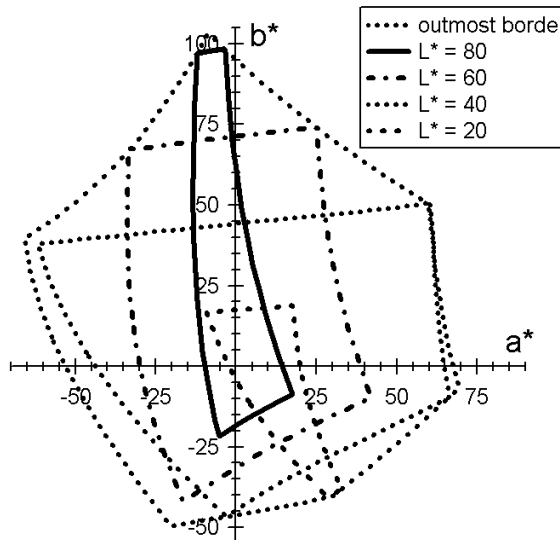

Figure 4. The a*b* diagram of a CMY offset printing process. For some L* values are cuts depicted. The dashed border line illustrates the maximal extend that can be reached for some L* values.
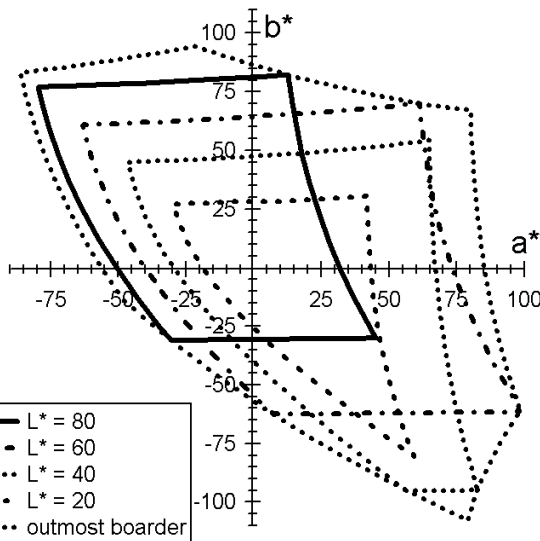

Figure 5. The a*b* diagram of a sRGB device. For some L* values are cuts depicted. The dashed border line illustrates the maximal extend that can be reached for some L* values.

In Figure 2b a scheme adequate to Figure 2a is presented. The highest L* value is received by lightning all three colors. If no color is present, the lowest L* value is received.

Applying the calculation scheme of the CMY color gamut accordingly an example plot can be given in Figure 5. It is calculated using the formalism of a sRGB device with a gamma of 2.4.

The CMYK Color Gamut

Most of the printers nowadays do not work with a CMY but with a CMYK color system. Therefore some investigations concerning the CMYK color gamut are needed.

It is desirable to extend the procedure found for CMY also to CMYK. The advantage of only three colors is that the cubes presented in Figure 2 are a good model to show the colors. Also color mixtures are relatively clear in their behavior. Adding a fourth color to the three will cause troubles. Color values are not further uniquely reachable. One expression of this are techniques for printing. Sometimes some amount of CMY colors is substituted by black color.

So first of all it has to be clarified what happens to the color gamut if a black color is added to CMY. Thinking in the CIE L*a*b* color space the black color will mainly decrease the value of L* (e.g. black L* = 11.5, a* = 0.1, b* = 0.7; all values given in this paper are measured in our environment). Looking at Figure 2a the upper three rhombi should still remain in the same position for the hull when adding black color. Adding black will decrease the L* value of the modified color. Consequently the modified color value with added black will fall inside of the color gamut body.

The situation for the lower three rhombi is somehow different. The bottom point of Figure 2a has an L* value of about 15.8 while the black color itself has a value of only 11.5. This means the pure black color has a lower L* value than CMY itself. All four colors (CMYK) printed one on each other will lower the L* value to about 5.3. Therefore a new bottom point must be CMYK instead of CMY. In consequence all the CMY hull values of the lower three rhombi will not remain the outside hull of the CMYK color gamut. Therefore the points of the lower three rhombi have to be changed in a special procedure.

First of all take a look onto the straight lines illustrated in Figure 2a, e.g. take the line from CM to CMY. The old step width used to calculate this line is to start with 100% cyan and 100% magenta and add yellow in steps of 10%. This leads to 11 points as indicated in Figure 3. The new end point that has to be reached is CMYK. Therefore it's easy to claim that starting at CM with 100% cyan and 100% magenta the new line to CMYK is calculated by adding the same amount of yellow and black in 10% steps at the same time. This will lead also to 11 points for this interconnection. Doing so each "new" point with added black instead of only yellow have a smaller L* value that adding just yellow. The result is that a line from CM to CMYK is created laying outside of the CMY color gamut. This can be done for all straight lines from the three secondary colors to CMYK.

The points laying in the areas bordered by these lines have to be treated in an other way. Please take a look onto Figure 6. In Figure 6a the step width of the colors is indicated for the CMY case. The increase of black can be done additionally as indicated in Figure 6b. Adding black to every single of these points will lead to a decrease of its L* value. Therefore all newly calculated points will lay outside of the CMY color gamut.

Doing the expansion of the color gamut form CMY to CMYK in this way it is sufficient to use also only 602 points for labeling the hull. For comparison: in the CMYK case there are $11^4 = 14,641$ possible color combinations when doing the mixing in 10% steps.

Having laid this basis all the other calculations presented before for the CMY color gamut can be applied.
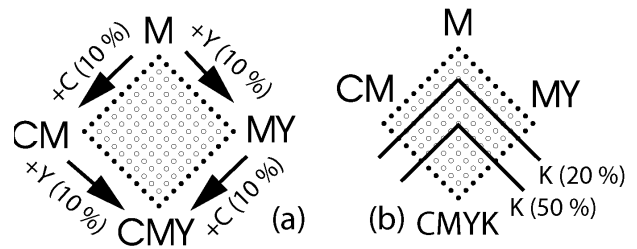

Figure 6. Section of Figure 3. Gradual color increase of (a) CMY and (b) CMYK.

Calculation of the Color Gamut Volume

The work done so far is a very good basis to calculate the volume of a color gamut. The hull is determined by a number of points. So slices at every L* value can be calculated. Calculating the volume of a color gamut can be subdivided into two parts: In the first part the gamut volume is divided into equidistant ΔL* parts. It is advisable to use a ΔL* of 1 or even lower. Then a number of slices can be calculated from the lowest to the highest L* value at the defined ΔL* step width. Then the area of every slice has to be calculated and all areas have to be summarized. The results of CMY, RGB and CMYK are presented in Table 1. The values obtained for CMY and CMYK color gamut volumes are in the same order of magnitude as results calculated for the SWOP color gamut volume of 307,195.[6]

Table 1. Volume calculation of Color Gamuts with different ΔL* values

	ΔL* = 10	ΔL* = 1	ΔL* = 0.1
RGB	819,304	819,294	819,303
CMY	341,477	339,844	339,859
CMYK	362,754	361,762	361,777

Conclusion

The presented work gives an approach to determine the color gamut of a device. This calculation is done isolated for a special device and not interconnected into color gamut mapping. Additionally the volume of a color gamut can be calculated.

For calculating the slices there might be the chance of some improvement for the accuracy. By choosing a better mathematical procedure the a*b* values can be determined more precisely than with the used linear regression method.

The advantage of our approach is that it can be used without applying a heavy mathematical background. Every point present on the hull will be used as it is. These points are neither boxed into any segmented sphere nor filled into any mathematical model.

Maybe this is exactly the disadvantage of this approach. It cannot be thought of a general application where it can be used to define a device's color gamut. Most procedures require a reduction of the number of sampling points that are needed. On the other hand it may be an adequate approach for some research studies when the number of color plots generated can be chosen freely.

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Biography

Andreas Willert received his diploma degree in experimental physics from the University of Kiel (Germany) in 1995 and finished his PhD studies at the Chemnitz Technical University (Germany) in 2000 specialized in ultra fast spectroscopy. Having spent three years in industry and information technology he started working at the Institute of Print and Media Technology at the Chemnitz University of Technology in 2003. His main working field is based in spectral color management.