Chromatic Opponency: Hypotheses and Psychophysical Performance

Claudio Oleari
Dipartimento di Fisica e Istituto Nazionale Fisica della Materia
Università di Parma, Parma, Italy

Abstract

The analysis of the Uniform Color System of the Optical Society of America (OSA-UCS) induces us to propose general hypotheses for defining color-opponency functions. The hypotheses are that the color signals is factorized into the product of the lightness with a pair of main chromatic opponency functions resulting from three processes: a linear transformation, a logarithmic compression and a chromatic opponency actuation. It results that the main chromatic opponency functions, individually with uniform scales, are equal to the logarithms of tristimulus-value ratios in a proper reference frame of the tristimulus space. The perceptual chromatic functions are a linear mixing of the main chromatic opponency functions. The performance of these hypotheses is successfully evaluated on the OSA-UCS system, for 10° visual field, and on the chromatic discrimination ellipses, for macular vision.

Introduction

In previous papers[1,2] it has been shown that the perceptual color opponencies in the Uniform Color System of the Optical Society of America (OSA-UCS),[3-6] represented by its coordinates (*j*, *g*), appear working in a mutually interacting way. In the hypothesis that such an interaction is due to a linear mixing of a pair of independent opponent mechanisms with scales satisfying a proper Weber's fraction, three chromatic opponency functions have been derived. Any pair of these three functions is a set of two independent functions with individually uniform scales. Moreover, the color signal is factorized into the product of the lightness with the chromatic opponency functions.[1,2,7]

This work reconsiders the color opponency by introducing very general hypotheses and the other guiding elements are simplicity and symmetry. Consequences of these hypotheses are the kind of signal compression and the opponency actuation. Moreover, the chromatic opponency functions are the logarithms of the ratios of two tristimulus values in a proper reference frame, named "main reference frame". The main reference frame can be defined because the perceptual chromatic functions, represented on constant lightness planes by geodesic lines, individually with uniform scales, exist. These hypotheses are strongly confirmed by the OSA-UCS system,[1,2] for extra macula vision, and by the chromatic discrimination ellipses,[8,9] for macular vision.

Hypotheses for Color Opponency Functions

The visual processing starts with the cone activations (L, M, S) and the final perceptual color is supposed specified by the lightness V and two perceptual chromatic functions (J, G).

The hypotheses for the color processing are:

I The lightness and the chromatic channels are supposed independent and parallel channels, and the color signals are the product of the perceptual chromatic functions with the lightness.

II The first linear transformation **T** transforms from the cone activation space (L, M, S) to a reference frame in the tristimulus space defined by three independent primary stimuli **A**, **B** and **C**. Let us call this *main-reference frame* and (A, B, C) *main tristimulus values*. Transformation **T** is typical of any given visual situation.

III The chromatic opponency, represented in the main reference frame and termed *main chromatic opponency*, is represented by a pair of the following three mutually dependent functions

$$\ln(A/B),\ \ln(B/C),\ \ln(C/A) \tag{1}$$

which are the simplest functions 1) *continuous* in the definition range, 2) *smooth*, 3) *antisymmetric* for the permutation of the two tristimulus values, 4) *zero-degree homogeneous* functions of the main tristimulus values (i.e. the chromaticity, hue and purity remain unaltered for any scale modification of the stimuli and consequently the Bezold-Brücke effect, which describes the hue shift as a function of the luminance, cannot be described).

IV The *perceptual chromatic functions* (J, G) are conventional and no particular perceptual chromatic opponency appears privileged. Their property is to represent, separately, uniform scales of colors. If the chosen pair of main chromatic opponency functions, e.g. $\ln(A/B)$ and $\ln(B/C)$, do not fit the perceptual chromatic functions, it is supposed that the perceptual chromatic functions are obtained by a linear transformation **C** on the chosen main chromatic opponency functions.

The perceptual chromatic functions (J, G) can be analytically converted into CIE chromaticity coordinates.

Complete symmetry exists among the three tristimulus values (A,B,C) as among the (L,M,S).

The scales of the main chromatic opponency functions (1) are separately uniform, because linearly related to the perceptual chromatic functions. Therefore the following Weber fractions hold true

$$\Delta(A/B)\ /\ (A/B) = k_{AB}$$

$$\Delta(B/C)\ /\ (B/C) = k_{AB} \quad (2)$$

$$\Delta(C/A)\ /\ (C/A) = k_{CA}$$

with $k_{AB} + k_{AB} + k_{AB} = 0$. If the Weber fractions (2) are satisfied, this means only that the quantities ln(*A*/*B*), ln(*B*/*C*) and ln(*C*/*A*), separately, have uniform scales and not necessarily the chromatic space has Euclidean metrics. Equations (2) hold for geodesic lines with constant lightness and the main reference frame can be found if constant lightness geodesics exist and are empirically defined.

Test of the Chromatic Opponency Hypotheses

The OSA-UCS System

The OSA-UCS system[3-6] is a color-appearance system for the CIE 1964 observer, whose color samples constitute uniformly spaced color scales belonging to six different conventional lines through each sample. These lines are termed geodesics. The grid lines constituted by the OSA-UCS coordinates (*j*, *g*) belong to this set of lines and are useful for testing globally the four hypotheses for the color opponency functions. Because a linear relation is supposed to exist between the main chromatic opponency functions ln(*A*/*B*), ln(*B*/*C*) and ln(*C*/*A*) and the perceptual chromatic functions, represented by the coordinates (*j*, *g*), the grid of the lines ln(*A*/*B*) = *constant* , ln(*B*/*C*) = *constant* and ln(*C*/*A*) = *constant*, represented on the (*j*, *g*) plane, must be constituted by three sets of equispaced parallel straight lines. All this is based on the main reference frame, if it exists. In such a case, the Weber fractions (2) must hold, where Δ(*A*/*B*), Δ(*B*/*C*) and Δ(*C*/*A*) are evaluated between contiguous samples belonging to the lines of the constant lightness lattices of the OSA-UCS system. The research for the main reference frame, based on a suitable parallelism index, is obtained by iteration technique. The chromaticity ***A***, ***B*** and ***C*** of the primaries, defining the main reference frame, are (Fig. 1)

$$\boldsymbol{A} = (x_{10}=0.9057,\ y_{10}=0.2391)$$

$$\boldsymbol{B} = (x_{10}=1.1134,\ y_{10}=-1.3384) \quad (3)$$

$$\boldsymbol{C} = (x_{10}=0.1604,\ y_{10}=-0.0258).$$

and are the same for all the lightnesses. The points ***A***, ***B*** and ***C*** are close to the confusion points but clearly distinct. The tristimulus values (*A*, *B*, *C*) are obtainable from the (X_{10}, Y_{10}, Z_{10}) ones by a linear transformation **T**

$$\begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} 0.65973 & 0.44916 & -0.10889 \\ -0.30528 & 1.21255 & 0.09273 \\ -0.03740 & 0.47951 & 0.55789 \end{pmatrix} \begin{pmatrix} X_{10} \\ Y_{10} \\ Z_{10} \end{pmatrix} . \quad (4)$$

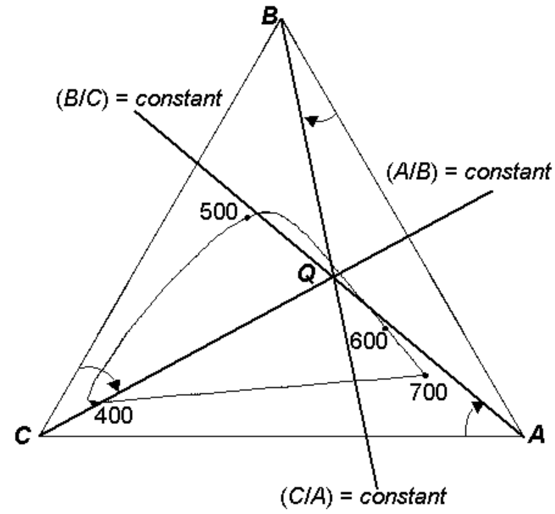

*Figure 1. CIE 1964 chromaticity diagram referred to the main reference primaries **A**, **B** and **C**, spectrum locus and straight lines radiating from the vertices **A**, **B** and* C *and crossing in the point **Q**=(A, B, C).*

The Weber fractions (2) are very well satisfied with a Root-Mean-Square value (RMS) generally lower than 1.5×10^{-4} and with a linear correlation index $|r| > 0.998$. (Fig. 2). Consequently, any pair of the main chromatic opponency functions (ln(*A*/*B*), ln(*B*/*C*)), or (ln(*B*/*C*), ln(*C*/*A*)) or (ln(*C*/*A*), ln(*A*/*B*)), supposed independent, can be considered as orthogonal coordinates, in which the constant lightness lattices of the OSA-UCS system have a grid of equispaced parallel straight lines (Fig. 3).

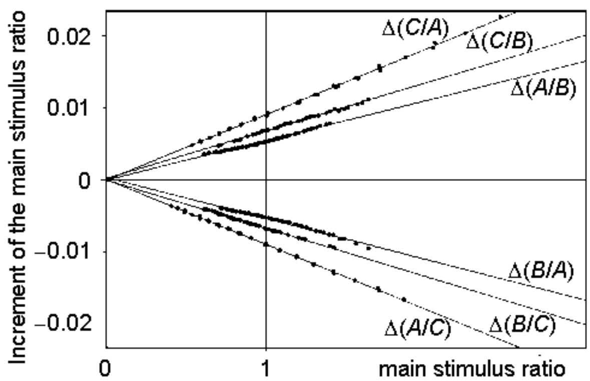

Figure 2. Lines associated to the Weber fractions related to the color samples of the OSA-UCS system at the lightness $L_{OSA} = 0$, fitting the points obtained from the OSA-UCS system.

The grid spacing is a linear function of the lightness with a correlation index $|r| > 0.9998$, an RMS < 0.16 for the *g* direction and an RMS < 0.07 for the *j* (Fig. 4), confirming the goodness of hypothesis I.

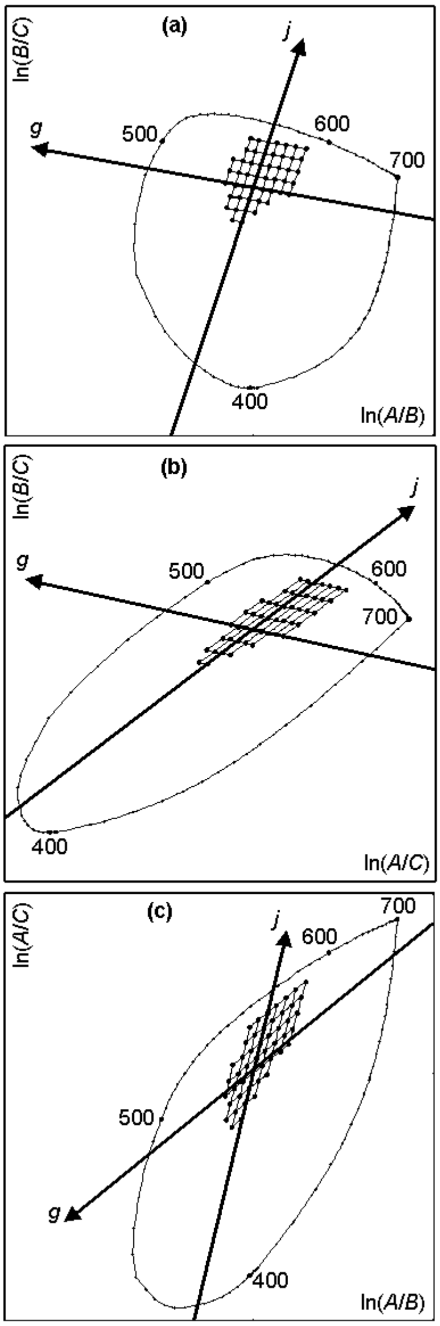

Figure 3. (a) (b) (c). *CIE 64 chromaticity diagram and the OSA-UCS lattice at* $L_{OSA} = 0$ *on coordinates* (ln(*A/B*), ln(*B/C*)), *on* (ln(*A/C*), ln(*B/C*)) *and on* (ln(*A/B*), ln(*A/C*)), *respectively.*

Finally, the grids of figure 3 can be transformed into square grids with reference axes parallel to the *j* and *g* directions by a suitable mixing of the main chromatic opponency functions. The mixing and lightness scaling for the pair of coordinates (ln(*A/B*), ln(*B/C*)) is

$$\begin{pmatrix} J \\ G \end{pmatrix} = \begin{bmatrix} S_J & 0 \\ 0 & S_G \end{bmatrix} \begin{bmatrix} -\sin\alpha & \cos\alpha \\ \sin\beta & -\cos\beta \end{bmatrix} \begin{pmatrix} \ln\left(\dfrac{A/B}{A_n/B_n}\right) \\ \ln\left(\dfrac{B/C}{B_n/C_n}\right) \end{pmatrix} \qquad (5)$$

where

- (J, G) are the coordinates corresponding to the OSA-UCS (j, g) and represent the perceptual color,
- $S_J = 2\ (0.5735\ L_{OSA} + 7.0892)$ and $S_G = -2\ (0.7640\ L_{OSA} + 9.2521)$ are suitable lightness-scale factors (Fig. 4);
- α and β are the angles between the reference axes of the main chromatic opponency functions and the directions of parallel lines with constant g and constant j, respectively, and $\sin\alpha = -0.1792$, $\cos\alpha = 0.9837$, $\sin\beta = 0.9482$ and $\cos\beta = 0.3175$ are good for all the lightnesses;
- the ratios (A_n/B_n) = 0.9366 and (B_n/C_n) = 0.9807 are ratios of the main tristimulus values related to the D_{65} white point and are introduced in order to fit the right origin of the OSA-UCS coordinates j and g.

The same holds for the other two pairs of main chromatic opponency functions, (ln(*B/C*), ln(*C/A*)) and (ln(*C/A*), ln(*A/B*)).

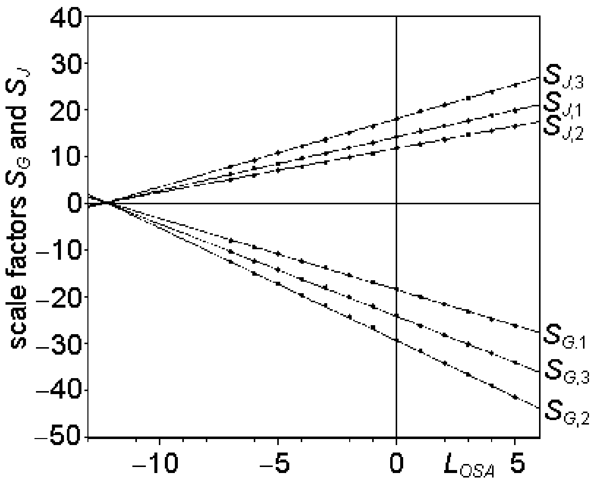

Figure 4. Scale factors $S_{G,i}$ *and* $S_{J,i}$ *as function of the lightness* (*i* = 1, 2, 3) *regards the pair of main chromatic opponency functions considered: i*=1 *is for* (ln(*A/B*), ln(*B/C*)), *i*=2 *for* (ln(*B/C*), ln(*C/A*)) *and i*=3 *for* (ln(*C/A*), ln(*A/B*)) *and the corresponding fitting lines. These lines are mutually crossing in a point with distance* 0.10 ± 0.22 *from the abscissa axis at lightness* L_{OSA} = –12.24 ± 0.12. *This crossing point defines the absolute zero of the lightness*

As expected, the diagrams, obtained from any pair of main chromatic opponency functions, after a transformation such as Eq. (5), are equal, i.e. the three pairs of main chromatic opponency functions are equivalent (Fig. 5). The coordinates (J, G) are very close to the corresponding coordinates (j, g) of the OSA-UCS system at any lightness and the grids are very close to square grids. The RMS between the coordinates (j, g) of the 424 OSA-UCS samples and the corresponding coordinates (J, G) computed by Eq. (5) for the three pairs of main chromatic opponency functions is 0.075 OSA-UCS units (1 OSA-UCS unit = 10 jnd).

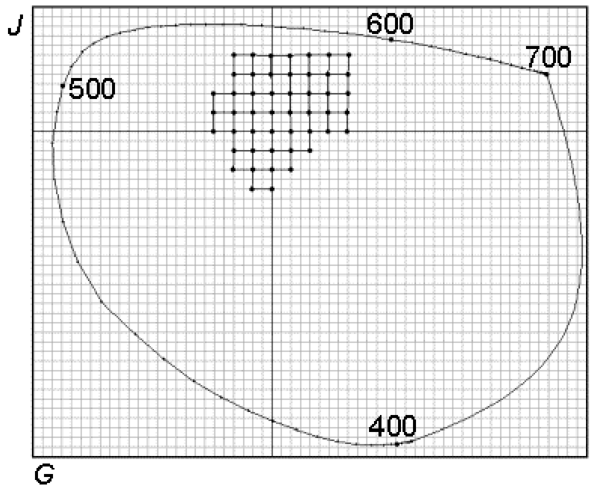

Figure 5. Plane $L_{OSA} = 0$ spanned by the coordinates (J, G) and the OSA-UCS lattice. The unit of distance represented by the grid is equal to 1 OSA-UCS unit that corresponds to 10 jnd. This same diagram is obtained in three different ways by proper dilatations and rotations from the diagrams represented in figure 3.

Chromatic Discrimination Ellipses

For the 2 degree observer, no lattice of geodesics with uniform scales of colors on constant lightness planes in tristimulus space exists for evaluating the goodness of the chromatic opponency hypotheses here proposed. The existing empirical data useful to define constant lightness geodesics could be the discrimination thresholds.[9] The available data at constant luminance are the chromatic discrimination ellipses.[10]

- which are affected by large uncertainty,
- depend on the individual observers,
- depend on the observing conditions, i.e. size of the matching field, luminance level and surround,
- are obtained by different techniques and therefore have different meaning.

Anyway, two sets of chromatic discrimination ellipses are here considered: the ellipses given by MacAdam[8] and those by Romero *et al.*[11] These two sets have different meanings and are obtained in different viewing situations.

MacAdam's ellipses are derived for the CIE 1931 observer and represent, with an uncertainty equal to 15%, one standard deviation in color matching at a luminance equal to 48 cd/m^2 for the test field and equal to 24 cd/m^2 for the white C surround. Let us do the approximate hypothesis that these ellipses in the uniform scale chromaticity diagram are represented by equal radius circles embedded in a square lattice. If we do a change of scale (dilatation) we alter such a square grid into a grid of equispaced parallel straight lines and the equal radius circles into equal ellipses. The directions of the semi axes of the ellipses are the same as the dilatation, then the procedure used to identify the main reference frame in the case of the OSA-UCS system is modified as follows by supposing that:

- at constant lightness the Weber fractions (2) hold for $\Delta(A/B)$, $\Delta(B/C)$ and $\Delta(C/A)$ evaluated on the semiaxes of the MacAdam ellipses,
- the MacAdam's ellipses, if represented on a plane spanned by a pair of the orthogonal coordinates ($\ln(A/B)$, $\ln(B/C)$), or ($\ln(B/C)$, $\ln(C/A)$) or ($\ln(C/A)$, $\ln(A/B)$), must appear as equal ellipses and with parallel axes, and therefore embedded into a grid of equispaced parallel straight lines.

The research for the main reference frame made by iteration technique as in the OSA-UCS system is successful, although with bigger uncertainty. The vertices of the three-component diagram ***ABC*** have chromaticity

$$\boldsymbol{A} = (x = 1.002,\ y = 0.091)$$

$$\boldsymbol{B} = (x = 1.513,\ y = -0.840) \qquad (6)$$

$$\boldsymbol{C} = (x = 0.162,\ y = -0.008)$$

and, as in the case of the OSA-UCS system, the points ***A***, ***B*** and ***C*** are close to the confusion points but clearly distinct. The tristimulus values (*A*, *B*, *C*) are obtainable from the (*X*, *Y*, *Z*) ones by a linear transformation **T**

$$\begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} 0.39094 & 0.67751 & -0.06845 \\ -0.09421 & 1.06609 & 0.02812 \\ 0.03233 & 0.31306 & 0.65462 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} . \qquad (7)$$

The Weber fractions (2) are well satisfied with an RMS $\approx 3.8 \times 10^{-3}$ and a linear correlation index $r \approx 0.96$. The ellipse with center (x = 0.160, y = 0.057) is not considered in this analysis, because its orientation is in contrast with the general setting of all the other ellipses. The MacAdam's ellipses, plotted on a plane spanned by any pair of the main chromatic opponency functions, considered as orthogonal coordinates, are almost equal and with parallel axes. The hypotheses II and III are confirmed together.

No empirical quantities considerable as perceptual chromatic functions, like the *j* and *g* coordinates of the OSA-UCS system exist in the case of MacAdam's ellipses. Therefore let us choose as perceptual chromatic functions the coordinates with the highest degree of uniformity, i.e. the ellipses are transformed to overlap equal radius circles with the lowest discrepancy. As seen before, the MacAdam's ellipses, plotted on a plane spanned by any pair of the main chromatic opponency functions, considered as orthogonal coordinates, are almost equal and with parallel axes. It induces us to perform a dilatation in the direction of axes of the ellipses transforming the ellipses into approximately equal radius circles. In this way we obtain three equal diagrams differently oriented. A rotation transforms these diagrams into the same orientation. Particularly we choose to set these diagrams with the long wavelength part of the spectrum locus parallel to the new abscissa axis. The dilatations are made with a suitable scale factor obtaining circles with radius equal to 1 jnd.

The complete transformation for the pair of main chromatic opponency functions $\ln(A/B)$ and $\ln(B/C)$ is

$$\begin{pmatrix} J \\ G \end{pmatrix} = \begin{pmatrix} -11.96064 & 22.68681 \\ -112.72260 & 0.58747 \end{pmatrix} \begin{pmatrix} \ln\left(\dfrac{A/B}{A_n/B_n}\right) \\ \ln\left(\dfrac{B/C}{B_n/C_n}\right) \end{pmatrix} \quad (8)$$

where

- J and G are the chromaticity coordinates with uniform scale, named in analogy with the OSA-UCS system,
- $(A_n/B_n) = 0.97326$ and $(B_n/C_n) = 0.90032$ are ratios of main tristimulus values related to the C white point and are introduced in order to define an origin of the main chromatic opponency functions.

The same holds, *mutatis mutandis*, for the other two choices of pairs of main chromatic opponency functions, ($\ln(B/C)$, $\ln(C/A)$) and ($\ln(C/A)$, $\ln(A/B)$).

At the end, as expected, the three diagrams obtained from the three pairs of main chromatic opponency functions, after a transformation such as Eq. (8), are exactly equal (Fig. 6), i.e. the three pairs of main chromatic opponency functions are equivalent. The uniformity of scale is good: the mean radius evaluated by 46 radii for each circle is 0.34 jnd (the expected value is 1/3 of jnd) with an RMS equal to 0.09. These results hold for the viewing situation of the MacAdam's ellipses.

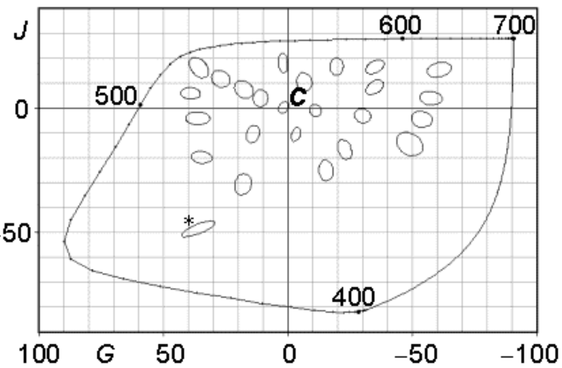

Figure 6. Uniform scale CIE 1931 chromaticity diagram (J, G) based on the MacAdam's ellipses. The origin ***C*** *corresponds to the standard white C and the unit of scale is the jnd. The starred ellipse is not considered in this analysis. The ellipses are enlarged 10 times.*

The ellipses given by Romero *et al.*[11] are considered for an additional test of the main chromatic-opponent functions. The Romero's ellipses, related to 20 stimuli at a luminance level of 12 cd/m^2 with dark surround and 3 observers, represent the constant luminance cross-elliptical sections of color-discrimination ellipsoids. With a testing procedure equal to the previous one we obtain an analogous diagram. The differences in the main reference frame and in the chromaticity diagram are due to the different meaning of these two data sets and to different fields surrounding the test field.

The hypothesis I cannot be checked by this analysis carried out on these chromatic-discrimination ellipses because these ellipses are given at constant luminance. Anyway this analysis confirms the global goodness of the hypotheses II and III for main chromatic opponency functions also for macular vision.

Conclusion

This work proposes a multi-stage color-vision model for the chromatic channel that correctly reproduces the psychophysical data, constituted by the OSA-UCS system for extra macula vision and by chromatic discrimination ellipses for macular vision.

The processes hypothesized are the following four: 1) a linear mixing **T** of the cone activations, 2) a logarithmic compression of the tristimulus values, 3) an antisymmetrization as the main chromatic opponency actuation, 4) a linear mixing **C** of the main chromatic opponency functions for fitting conventional perceptual chromatic functions.

The linear transformation **T** in the tristimulus space depends on the adaptation and this dependence is unknown. The knowledge of the **T**-matrix elements as functions of the complete visual situation could explain the color-appearance phenomena and the Bezold-Brücke hue shift ignored by the chromatic opponency hypotheses here proposed.

A complete model of the color-vision processing needs a lightness channel that is not considered here.

Acknowledgments

This work has been supported by the National Scientific Research Program “Cofinanziamento del MIUR 2002” titled “Modellizzazione psicofisica e computazionale della percezione contestuale del colore ”.

BASIC-language routines for the transformations between the (X_{10}, Y_{10}, Z_{10}) and (L_{OSA}; J, G) coordinates and between (x, y) and (J, G) can be obtained from claudio.oleari@unipr.it .

References

1. Oleari C. Color Opponency and Scale Uniformity in the OSA-UCS System: the Geometrical Structure. In: AIC 2001, The 9th Congress of the International Colour Association. Chung R and Rodrigues A, editors. Vol. 4421. Washington, DC: SPIE; 2002. p 852-855.
2. Oleari C. Color Opponencies in the system of the uniform color scales of the Optical Society of America, J. Opt. Soc. Am. A 2004; 21: in press.
3. MacAdam DL. Uniform Color Scales. J Opt Soc Am 1974; 64, 1691-1702.
4. MacAdam DL. Colorimetric data for samples of OSA uniform color scales. J Opt Soc Am 1978; 68, 121-130.
5. MacAdam DL. Color Measurement. p 165-177. Berlin: Springer-Verlag; 1985.
6. Nickerson D. OSA Uniform Color Samples: A Unique Set. Color Res Appl 1981; 6: 7-33.
7. Oleari C. Uniform-color space for 10° visual field and OSA uniform-color scales. J Opt Soc Am A 1993; 10: 1490-1498.
8. MacAdam DL. Visual sensitivities to color differences in daylight. J Opt Soc Am 1942; 32: 247-274.

9. Oleari C. Uniform scales of chromaticity and hidden geometrical structure of the color opponencies in foveal vision. In: AIC Color 2002 SI, Color & Textiles, Book of Proceedings, Maribor Slovenia August 29-31 2002. p 25-28.
10. Wandell BA. Color measurement and discrimination. J Opt Soc Am A 1985; 2: 62-71.
11. Romero J, Garcia JA, Jiménez del Barco L. Hita E. Evaluation of color-discrimination ellipsoids in two-color spaces. J Opt Soc Am A 1993; 10: 827-837.

Biography

Claudio Oleari obtained the doctoral degree in Physics at Padua University in 1969. Today he is Associated Professor of General Physics at Parma University.

He has been coordinator for the Colorimetry Group in the Italian Society of Optics and Photonics.

The main results of his recent scientific activity in colorimetry and color vision are: 1) Methamerism-invariance group; 2) uniform scale chromaticity diagram; 3) new formula for the OSA-UCS system; 4) logarithmic chromatic response functions; 5) experimental definition of the dichromatic confusion points; 6) interobserver comparison of color-matching functions; 7) spectral data deconvolution; 8) the book: “Misurare il colore”, U. Hoepli Editore - SIOF, Milano (1998).