Computational Model for Chromaticity Differences

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Abstract

In this work we create a computational model, which assesses chromaticity differences based on an ellipse data set. The used ellipse data sets are the MacAdam ellipses in the CIE 1931 (x,y)-chromaticity diagram and the ellipses which were used to derive the CIEDE2000 color-difference formula in the CIELAB color space. In general the ellipse data set can be any set of planar chromaticity ellipses. The chromaticity differences are calculated from the surfaces which are defined by the ellipse data set and the two chromaticity points whose the chromaticity difference is calculated. The distances along the surfaces are calculated by a method based on the Weighted Distance Transform On Curved Space (WDTOCS). The computational model corrects the planar values in chromaticity difference calculation.

1. Introduction

Color-difference formulas are mainly based on studies, where chromaticity differences are measured on various illumination levels. Color differences are derived from chromaticity-difference measurements using weighting functions and parametric factors. This has lead to several quite complicated color-difference formulas with numerous factors [1]. The latest CIE recommended color-difference formula, CIEDE2000, was developed with a set of variables for the parametric correction of the error from the CIELAB ΔE formula [2]. The CMC model for textile industry is dividing the (a^*b^*)-plane into microfacets thus compensating the planar color difference errors [3].

Our purpose was to develop a simple computational model, which gives an equal perceived chromaticity difference in every part of the CIELAB color space for equal chromaticity differences. In our previous works we have developed a model based on surfaces [4, 5], but due to it's shortcomings an advanced model is required.

The paper is organized as follows: in Chapter 2 we define the computational model for chromaticity differences. Experiments are in Chapter 3 and in Chapter 4 we draw the conclusions.

2. Computational Model

The chromaticity differences are calculated from the surfaces which are defined by the chromaticity-difference ellipse data set and by the two chromaticity points (x_0, y_0) and (x_1, y_1), whose chromaticity difference is under consideration. For each chromaticity difference a surface is created and the surface consists of a chromaticity-difference grid, i.e. a grid is created to surround the first chromaticity point (x_0, y_0), denoted as a starting point. A chromaticity difference is calculated from the starting point to all other points in the grid. The total chromaticity difference is calculated by WDTOCS [4] summing up the chromaticity differences on the shortest path along the surface between the two chromaticity points (x_0, y_0) and (x_1, y_1). In this manner the local variance of ellipse parameters are taken into account.

2.1. Defining the Surface

The definition of a surface bases on the three parameters of an ellipse: 1-2) the lengths of a- and b-semi-axes and 3) the rotation angle θ from the x-axis. The lengths of the semi-axes define the just-perceptible chromaticity differences to the semi-axes directions on the corresponding point. The length of just-perceptible chromaticity difference to any direction, c-axis, can be calculated using ellipse parametrization.

The total chromaticity difference is thought to be multiple of a reference axis r, which is defined to be equal as the longest just-perceptible chromaticity difference in the used data set. The rest of just-perceptible differences are parallel projections of the reference axis, which are obtained by vertical rotation of the reference axis. The height of each point of the surface can be calculated using both the distance d_i between the starting point (x_0, y_0) and a chromaticity point (x_i, y_i) in the chromaticity-difference grid and the angle of the vertical rotation α in a right-angled triangle, see Figure 1.

Each point in the chromaticity-difference surface is defined as follows. Let $\mathcal{C}$ denote the chromaticity-difference surface and (x_0, y_0) be the starting point. Each point i in

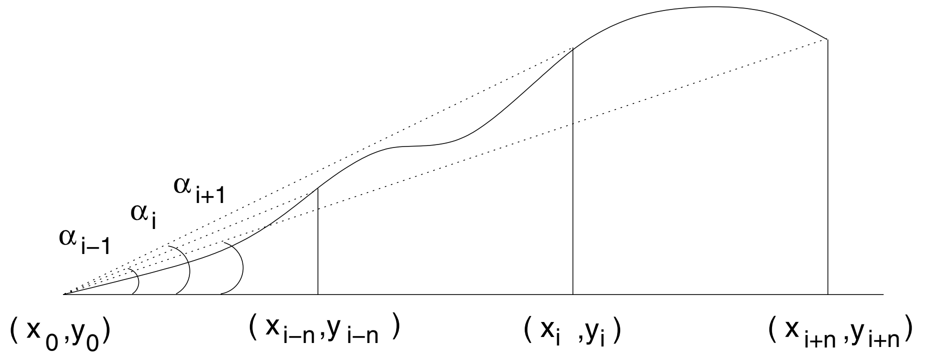

Figure 1: The definitions of the height of the each point in the surface. The points (x_{i-n}, y_{i-n}) and (x_{i+n}, y_{i+n}) denote points, which are calculated before and after of the point (x_i, y_i), respectively.

the $\mathcal{C}$ is defined as

$$\mathcal{C}(i) = d_i \cdot \tan(\alpha), \tag{1}$$

where

$$d_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2},$$

$$\alpha = \arccos \frac{c_i}{r},$$

(x_0, y_0) is the starting point, (x_i, y_i) a chromaticity point in the chromaticity-difference grid, r the reference axis and

$$c_i = \sqrt{(a_i \cdot cos(\beta))^2 + (b_i \cdot sin(\beta))^2},$$

where

$$\beta = \arctan(\frac{a_0}{b_0} \cdot \tan(\varphi)),$$

a_i and b_i the corresponding ellipse semi-axes, a_0 and b_0 the semi-axes of ellipse in the starting point (x_0, y_0) and φ the angle between the a-semi-axis and the line connecting the chromaticity points, see Figure 2.

The distances along the surface are calculated by the Weighted Distance Transform on Curved Space (WDTOCS). The total chromaticity difference is calculated as follows

$$\Delta E_c = \frac{D_s}{r} \tag{2}$$

where D_s is the shortest calculated distance between the two chromaticity points (x_0, y_0) and (x_1, y_1) along the surface and r is the reference axis.

Our previous model, the Enhanced Model for Chromaticity Differences [5] combined the calculated chromaticity differences from the surfaces which were oriented to $a - direction$ and to $b - direction$ in order to assess chromaticity difference to any direction. This lead to problem that the chromaticity difference was never calculated to

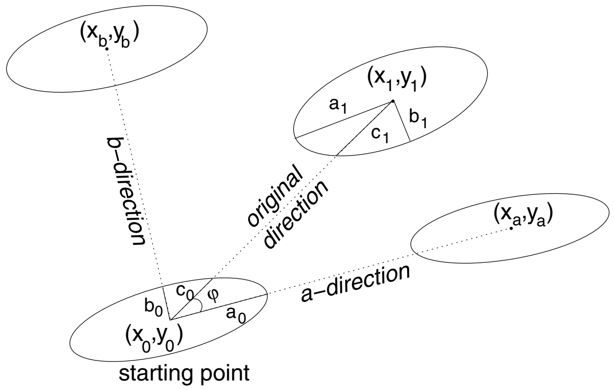

Figure 2: The definitions of $a - direction$ and $b - direction$ on the xy-plane.

its original direction, see Figure 2. The disadvantage was solved using the ellipse parametrization before the creation of the surface to achieve the length of c_1-axis, which is the just-perceptible chromaticity difference on the original direction.

2.2. Ellipse Data Sets

In this work the ellipse data sets were MacAdam ellipses [6] and ellipses which were used in deriving the color difference formula CIEDE2000 [2], in this article denoted as CIEDE2000 data set. The MacAdam data set was used in verification of the computational model, because the interpolation of the MacAdam ellipses is straightforward: they do not overlap each other and they form a harmonic set.

The CIEDE2000 data set consists of several different studies. Luo et al. combined four different color discrimination data sets: BFD-perceptibility [7], RIT-DuPont [8], Leeds [9] and Witt [10] data sets to one single data set.

CIEDE2000 data set consists of 116 ellipses, which lie in different illumination levels. The orientations and sizes of the ellipses do not a form harmonic set and the ellipses close in the illumination level are overlapping each other. This makes the interpolation of ellipse parameters problematic. The interpolation of the ellipse parameters was made in the Matlab environment using the nearest-neighbor interpolation method. The parameters were interpolated to $L^*a^*b^*$ color space in order to define ellipse parameters in every point in the color space.

The four data sets, from where the CIEDE2000 data set was derived, consist of different number of ellipses, which cause imbalance among the data sets in terms of influence to CIEDE2000 data set. For example, BFD-perceptibility data set consists of 81 chromaticity difference ellipses compared to 6 ellipses from Witt data set. Between the Witt and BFD-perceptibility data sets, the influence of the BFD-perceptibility data set is remarkably stronger to CIEDE2000 data set. In this work the data sets are taken as they are without any weighting.

3. Experiments

The experiments were performed using two different data sets. First the model was tested with MacAdam ellipses to verify the computational model, and the results were contrasted with the previous model. In the second experiment measurements were made with the CIEDE2000 data set and the obtained results were compared with the results from the CIEDE2000 color-difference formula.

3.1. Experimental Results from the MacAdam Data Set

The results from the MacAdam data set show the influence of the model to measured chromaticity difference compared to the Euclidean distance on the (x,y)-plane. In Table 3.1 the chromaticity differences are calculated from five MacAdam ellipses. The chosen ellipses are enumerated in Figure 3. Chromaticity differences are calculated from the center of the ellipse to the edge of the ellipse. The results as a difference between the two chromaticity points should be a constant value 1.0 and it is then comparable to the standard deviation.

Table 3.1 also presents results from our previous model [5] for chromaticity difference calculations from the same ellipses. The new model achieved considerably more accurate results than the previous model was able to achieve. The chromaticity differences assessed by the previous model vary between 0.94 and 1.08. However, the variance of the chromaticity differences calculated by the new model is from 0.997 to 1.017. The improvement can be clearly seen especially when the angle φ is not $0°$ or $90°$, when the previous model outperforms worst.

Also the variation of chromaticity differences assessed by the CIEDE2000 vary significantly more than the ones from the new computational model.

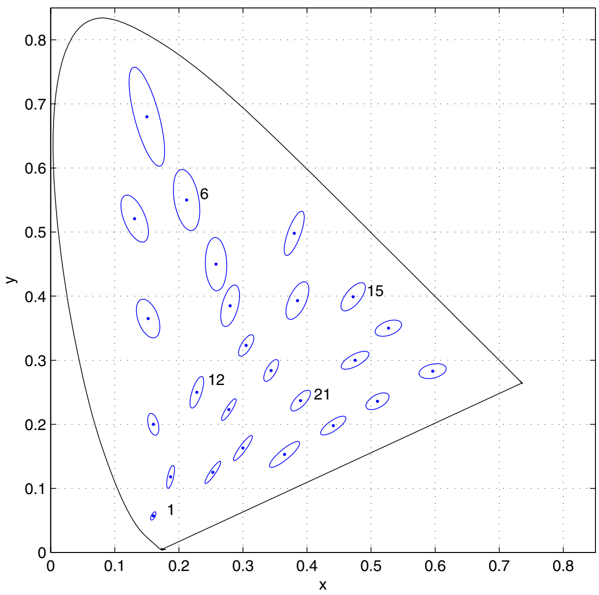

Figure 3: The numbers of the chosen ellipses. The axes of plotted ellipses and the chromaticity differences are 10 times their actual lengths.

3.2. Experimental Results from the CIEDE2000 Data Set

The second experiments deals with the chromaticity difference calculations using CIEDE2000 data set. The achieved results are compared to chromaticity differences calculated by CIEDE2000 color-difference formula [2]. Previous experiments confirmed the performance of the calculation model, but now the interpolation of the ellipse parameters in the CIEDE2000 data set was more challenging.

In Figure 4 the chromaticity differences are calculated from the ellipses of CIEDE2000 data set. The chromaticity differences assessed by our model are marked with black line and denoted as ΔE_c. The differences are calculated from the center of the ellipse to the edge of the ellipse. The calculated chromaticity difference should equal to 1.0 and it is then comparable to the just-perceptible chromaticity difference. The reference white in the calculations was X_0 = 94.811, Y_0 = 100.000, Z_0 = 107.304.

The chromaticity differences are calculated from four different regions: 1) white area near origin (ellipse #1), 2) blue area on the lower half of the a^*b^* space (ellipse #2), 3) red area on the upper half of the a^*b^* space (ellipses #3 and #4) and 4) green area on the left half of the a^*b^* space (ellipses #5 and #6).

From Figure 4 it can also be observed the assessed chromaticity differences by CIEDE2000 color-difference formula and the differences are marked with wide grey line and denoted as ΔE_{00}. The color-difference formula was used in chromaticity difference calculations excluding

Table 1: ***Chromaticity differences from the MacAdam ellipses.***

number of the ellipse	angle φ	distance on the (x,y)-plane	chromaticity difference ΔE_c	chromaticity difference from the previous model [5]	chromaticity difference from CIEDE2000 ($L^* = 50$)
1	0.0	0.00085	1.0056	1.00	0.21
1	90.0	0.00035	1.0099	0.98	0.13
6	43.5	0.00310	1.0167	0.95	0.29
6	76.2	0.00270	1.0058	1.08	0.25
12	17.7	0.00210	1.0070	0.94	0.58
12	57.9	0.00100	1.0061	0.97	0.54
15	0.0	0.00320	1.0052	0.97	0.57
15	90.0	0.00140	1.0016	0.97	0.30
21	0.0	0.00250	1.0034	0.95	0.31
21	90.0	0.00100	0.9979	0.95	0.15

the illumination differences.

On the white area our model performs reasonably well compared to CIEDE2000. In general between the CIEDE2000 and the new computational model, the results of the new model vary less. In the ellipse #5 the variance of the results is greater than in the other ellipses, nevertheless the variance is less than 10 %. In fact in the ellipse #5 the results from the CIEDE2000 and our model are the closest to each other.

4. Conclusions

A computational model for chromaticity differences is defined. The model is based on the surface, which are defined by the chromaticity-difference ellipse data set and the two chromaticity points from which the chromaticity difference is calculated. The distances are calculated by the Weighted Distance Transform on Curved Space. The surface varied according to the two chromaticities, whose difference was under consideration.

The achieved results were promising. The accuracy of the calculations from the MacAdam data set were improved remarkably and the new model could overcome the disadvantages of the original Enhanced model for chromaticity differences [5]. The experimental results validated the computational model for chromaticity differences.

The CIEDE2000 data set caused new problems in the chromaticity difference calculations. The interpolation of ellipse parameters of the CIEDE2000 data set was quite challenging, but nevertheless the interpolation was generally successful.

The results show that the new model could assess chromaticity differences reasonably well compared to CIEDE-2000. There are some occurrences, where the CIEDE2000 color-difference formula gave notably dissimilar results than our model. These occasions have to be investigated more specifically, because the CIEDE2000 formula is widely tested and adjusted to small scale color differences.

The obtained results were encouraging. With further study of interpolation methods and data sets better results could be achieved. Also the development of color-difference model will be an essential part of future work.

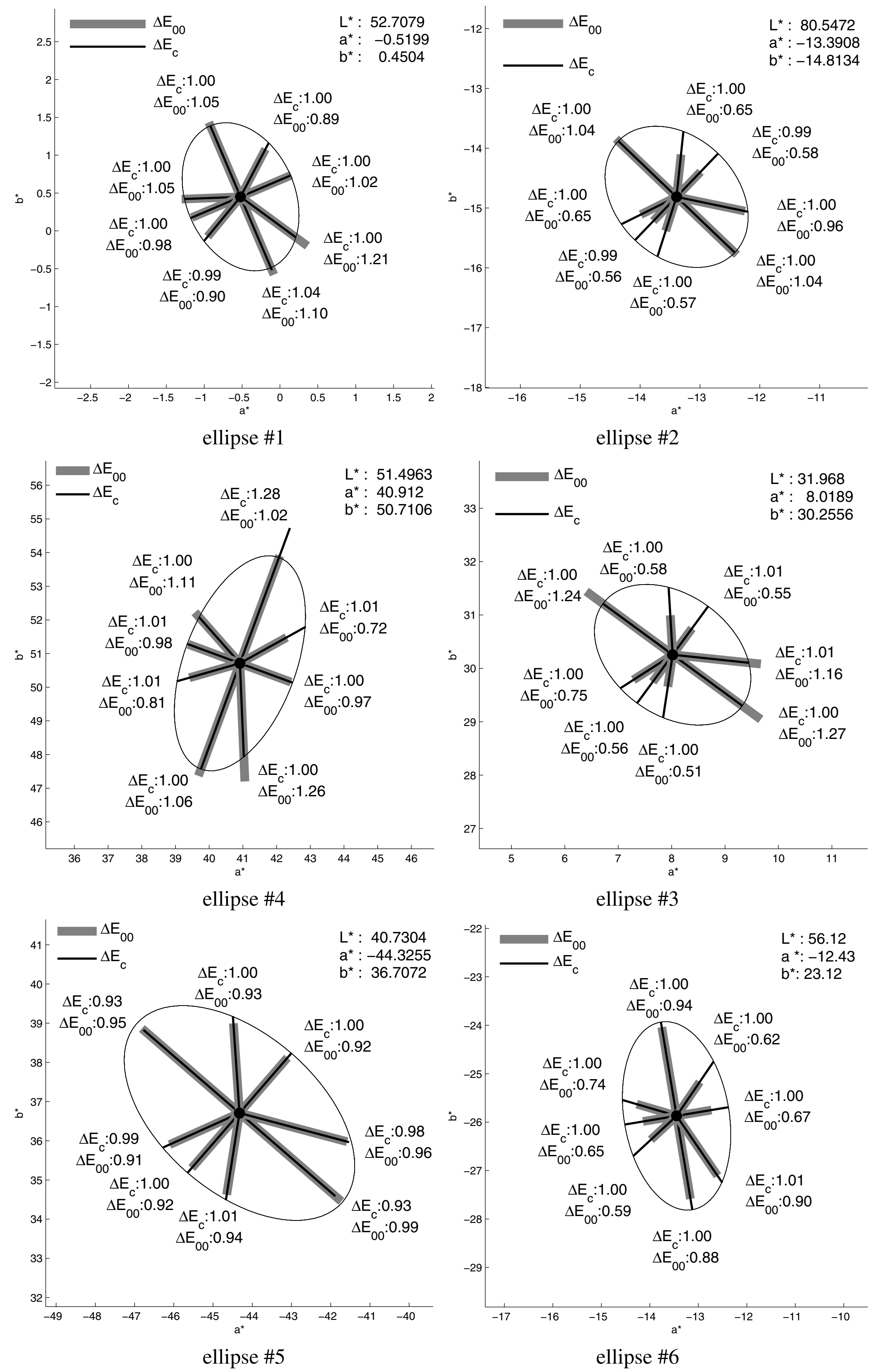

Figure 4: *The ellipses showing the relative chromaticity differences assessed by this work and CIEDE2000 color-difference formula (marked with wide black line and grey line and denoted as ΔE_c and ΔE_{00} respectively).*

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Biography

Toni Kuparinen is a MSc. student in Lappeenranta University of Technology. His main research interests are in color science and spectral image processing.