Pigment Particles and Their Effect on the Colour Gamut of Printing

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Abstract

This paper is concerned with changes in the colour gamut when overprinting a substrate with colour ink layers. The deviations of the resulting colour are dependent on the thickness, size and colour of pigment particles and their volume concentration. If the colour ink layers are sufficiently thin then it is shown that the substrate also influences the final result. All the pigment particles are non-transparent and the effect of spectral reflectances are also considered.

Introduction

This paper will show how one can better understand the control of colour gamut in printing process. The first section concerns the simplest case – transmittance of one colour, then the theory is extended to consider two ink layers. The related question of colour co-ordinates follows and the wet on wet case is also analyzed. Finally a sample calculation is given and future work indicated. Based on the analogy suggested by Kubelka and Munk[1] two light portions are considered in this investigation. Two technologies of printing will be analysed in this paper: "wet on dry" and "wet into wet." In the first case the second ink layer is printed on the dry layer and in the second case into the wet layer. Emmel and Hersch[2] have solved the light scattering and ink spreading phenomena by the transfer matrix, which consists of two elements: K (the light absorption coefficient) and S (the light scattering coefficient of the medium). Their treatment is mainly focused on dot gain analysis. The same problem has been solved by Rogers[3] using the exponential line-spread function. Agar[4] in his study deals with a high-resolution halftone microstructure image assuming dot profiles with Gaussian roll-offs to get a high resolution transmission image. The point spread function of the paper substrate is a Gaussian function. Colour gamut of halftone colour depends on an optical dot gain which was first studied by Yule and Nielsen.[5]

Transmittance of a One Colour Ink Layer

Let us consider the light flux passing through a layer of thickness t with volume concentration N of pigment particles. The distribution function of area S_i of pigment particles will be denoted p(s). The mean value of the area of pigment particles is given by Eq. 1.

$$S = \sum p_i S_i \tag{1}$$

The loss of light flux dΦ in a layer of thickness dx we obtain Eq. 2, see Fig. 1.

$$d\phi = -\phi(NS + \varepsilon)dx \tag{2}$$

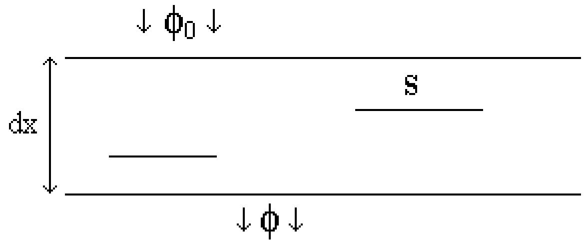


Figure 1. Transmission of light through a colour ink layer

We derive Eq. 3 after integration.

$$\phi = \phi_o e^{-(NS+\varepsilon)t} \tag{3}$$

$$T = \phi/\phi_o = e^{-(NS+\varepsilon)t} \tag{4}$$

The transmittance is given by Eq. 4 based on analogy with the Lambert-Beer law. Furthermore, we shall assume that there is no dispersed dye in the layer that means $\varepsilon = 0$. Let us now establish coverage P of an area a^2 of substrate, which is dependent on the thickness of the layer and the volume concentration of the pigment particles. We will now introduce two non-dimensional (relative) quantities the diameter and area of the pigment particle:

$$\xi = d/a \qquad s = \pi\xi^2/4 = S/a^2 \tag{5}$$

The exponent of Eq. 3 can be written in the form where:

$$NSt = ns = \mu t. \tag{6}$$

When describing the self-shielding (self-shadowing) of particles we can assume that the opaqueness is given by the value $T_i = 0$. The transmittance T of the layer and coverage P of the substrate is given by Eq. 7 in which n represents the number of particles in the layer:

$$T = exp\ (-ns) \qquad P = 1 - T \tag{7}$$

Transmittance of Two Ink Colour Layers

Let us now consider the change of light flux through two sufficiently thin layers see Fig.2.

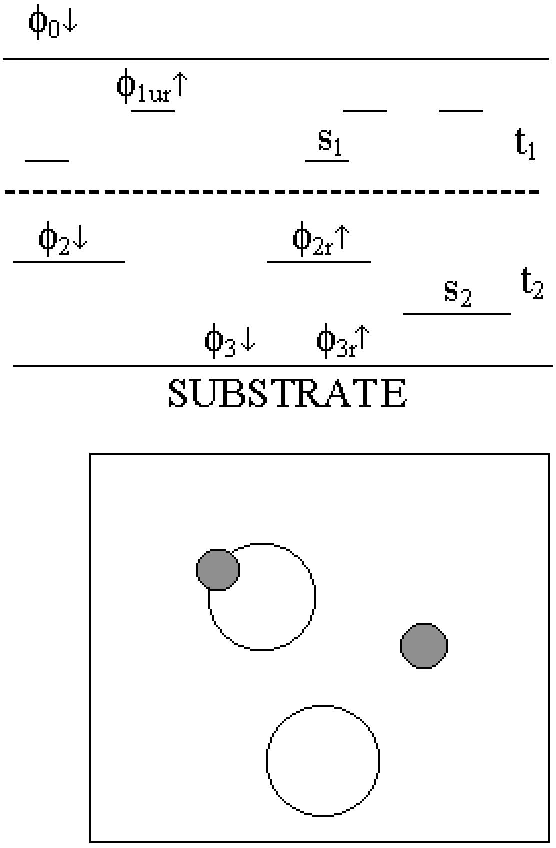


Figure 2. The coverage P_1 by the ink layer with areas s_1

The coverage of the substrate with individual colours and resulting transmission is represented by Eq. 8, 9, 10:

$$P_1 = 1\text{- }\exp(-n_1 s_1) = 1\text{-}T_1 \tag{8}$$

$$P_2 = 1\text{- }\exp(-n_2 s_2) = 1\text{-}T_2 \tag{9}$$

$$T = \exp(-n_1 s_1 - n_2 s_2) = T_1 T_2 \tag{10}$$

then Φ is the light flux striking the substrate. Fig. 3 shows an example in which every layer has 250 particles with relative areas $s_1 = 0.05$ and s_2=0.1. The length of the line $WC_{1u} = T_1$ and $WC_{2u} = T_2$ corresponds to the transmission of individual layers. The total transmission equals

$$T = T_1 T_2 = WC_{12} \tag{11}$$

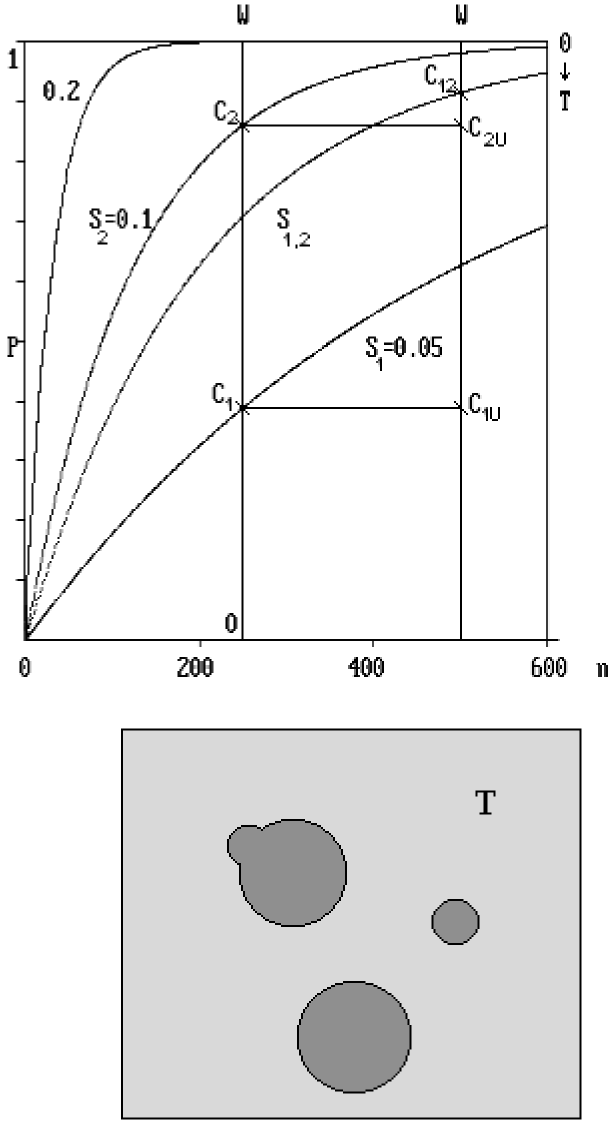


Figure 3. The transmittance of two ink layers

In Fig. 3 we can see ”knee” saturation. Above this value coverage of the substrate grows only marginally and requires a notable quantity of pigment. The coloured layers may by printed in sequence. The outer layer has s_1 and inner layer has s_2 and vice versa.

Case a) The Outer Layer with Particle Area s_1

The light flux is broken into three reflections. Part of this beam is reflected by particles in the outer layer Φ_{1ur}, another part in part in the inner layer Φ_{2r} and rest Φ_{3r} is reflected by the substrate see Fig. 2. Secondary reflections have been ignored because the magnitude of the flux rapidly drops to zero, which is in agreement with Fig. 3 and represented by the flux, described by Eq. (12), (13), (14), (15).

$$\phi_{1ur} = \phi_0 P_1 = \phi_0(1\text{-}T_1) \tag{12}$$

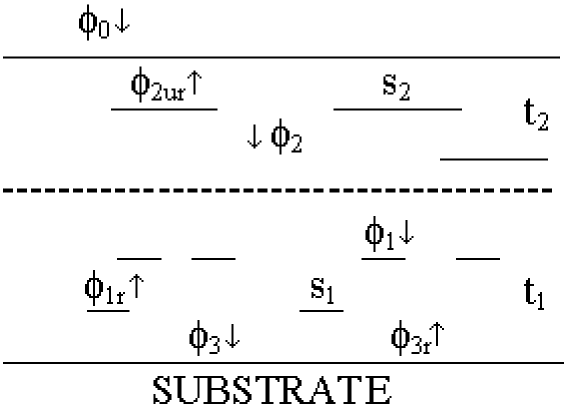
$$\phi_2 = \phi_0 - \phi_{1ur} = \phi_0(1-P_1) = \phi_0 T_1 \quad (13)$$

$$\phi_{2r} = \phi_2(P_2 - P_1 \cap P_2) = \phi_0 T_1 \cap (1-T_2) \quad (14)$$

$$\phi_3 = \phi_0 T_1 \cap T_2 = \phi_0 T \quad (15)$$

Case b) The Outer Layer with Particle Area s_2

By analogy with case a) the flux can be shown as written in Eq. (16), (17), (18), (19). This situation can be shown in Fig. 4.



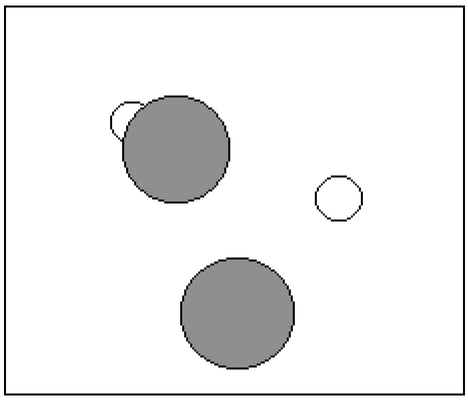


Figure 4. The coverage P_2 by ink layer with areas s_2

$$\phi_{2ur} = \phi_0 P_2 = \phi_0(1-T_2) \quad (16)$$

$$\phi_1 = \phi_0 - \phi_{2ur} = \phi_0(1-P_2) = \phi_0 T_2 \quad (17)$$

$$\phi_{1r} = \phi_1(P_1 - P_1 \cap P_2) = \phi_0 T_2 \cap (1-T_1) \quad (18)$$

$$\phi_3 = \phi_0 T_1 \cap T_2 = \phi_0 T \quad (19)$$

Calculation of the Colour Co-Ordinates CIExy

Given the area of the particle coverage in an individual layer we can determine the tristimulus values X, Y, Z and therefore calculate colour co-ordinates. If the pigment particles have spectral reflectances $r_1(\lambda)$ and $r_2(\lambda)$ then the transmittance is expressed by Eq. 20.

$$T_i = \exp(-n_i s_i) = \exp(-\mu_i t_i) \equiv e(\mu_i t_i) \quad (20)$$

If the sequence of printed colours is as shown in case a) we derive the values X_1, Y_1, Z_1 and constant k_{11} by using Eq. 12, 13, 14, 15 see Eq. 21.

$$\begin{aligned} X_1 &= \int \phi_0[1-\exp(-\mu_1 t_1)]\, r_1(\lambda)x(\lambda)d\lambda = \\ &= k[1-\exp(-\mu_1 t_1)] \int P_e(\lambda)\, r_1(\lambda)\, x(\lambda)d\lambda = k_{11} X_1' \\ Y_1 &= \int \phi_0[1-\exp(-\mu_1 t_1)]\, r_1(\lambda)y(\lambda)d\lambda = \\ &= k[1-\exp(-\mu_1 t_1)] \int P_e(\lambda)\, r_1(\lambda)\, y(\lambda)d\lambda = k_{11} Y_1' \\ Z_1 &= \int \phi_0[1-\exp(-\mu_1 t_1)]\, r_1(\lambda)z(\lambda)d\lambda = \\ &= k[1-\exp(-\mu_1 t_1)] \int P_e(\lambda)\, r_1(\lambda)\, z(\lambda)d\lambda = k_{11} Z_1' \end{aligned} \quad (21)$$

where $k_{11} = k\,[1-\exp(-\mu_1 t_1)]$. and $P_e(\lambda)$, $x(\lambda)$, $y(\lambda)$, $z(\lambda)$ stand for the spectral radiance and tristimulus functions.

The second (inner) colour layer is described as shown in Eq. 22.

$$\begin{aligned} X_2 &= k\exp(-\mu_1 t_1)][1-\exp(-\mu_2 t_2)] \int P_e(\lambda)\, r_2(\lambda)\, x(\lambda)d\lambda = k_{12} X_2' \\ Y_2 &= k\exp(-\mu_1 t_1)][1-\exp(-\mu_2 t_2)] \int P_e(\lambda)\, r_2(\lambda)\, y(\lambda)d\lambda = k_{12} Y_2' \\ Z_2 &= k\exp(-\mu_1 t_1)][1-\exp(-\mu_2 t_2)] \int P_e(\lambda)\, r_2(\lambda)\, z(\lambda)d\lambda = k_{12} Z_2' \end{aligned} \quad (22)$$

where $k_{12} = k\exp(-\mu_1 t_1)][1-\exp(-\mu_2 t_2)]$

For the case of specular reflectance from the substrate we obtain Eq. 23.

$$\begin{aligned} X_3 &= k\exp(-\mu_1 t_1 - \mu_2 t_2)] \int P_e(\lambda)\, r_3(\lambda)\, x(\lambda)d\lambda = k_{13} X_3' \\ Y_3 &= k\exp(-\mu_1 t_1 - \mu_2 t_2)] \int P_e(\lambda)\, r_3(\lambda)\, y(\lambda)d\lambda = k_{13} Y_3' \\ Z_3 &= k\exp(-\mu_1 t_1 - \mu_2 t_2)] \int P_e(\lambda)\, r_3(\lambda)\, z(\lambda)d\lambda = k_{13} Z_3' \end{aligned} \quad (23)$$

where $k_{13} = k\exp(-\mu_1 t_1 - \mu_2 t_2)]$ and $r_3(\lambda)$ means spectral reflectance of the substrate. The resulting tristimulus colour values are given by Eq. 24 and colour co-ordinates are given by Eq. 25.

$$\begin{aligned} X &= X_1 + X_2 + X_3 = k_{11} X_1' + k_{12} X_2' + k_{13} X_3' \\ Y &= Y_1 + Y_2 + Y_3 = k_{11} Y_1' + k_{12} Y_2' + k_{13} Y_3' \\ Z &= Z_1 + Z_2 + Z_3 = k_{11} Z_1' + k_{12} Z_2' + k_{13} Z_3' \end{aligned} \quad (24)$$

$$\begin{aligned} x &= X/(k_{11}V_1 + k_{12}V_2 + k_{13}V_3) \\ y &= Y/(k_{11}V_1 + k_{12}V_2 + k_{13}V_3) \end{aligned} \quad (25)$$

where

$$\begin{aligned} V_1 &= X_1' + Y_1' + Z_1' \\ V_2 &= X_2' + Y_2' + Z_2' \\ V_3 &= X_3' + Y_3' + Z_3' \end{aligned}$$

For the case of diffuse reflectance of the substrate k_{13} equals $\exp 2[(-\mu_1 t_1 - \mu_2 t_2)]$ applies. If the colour sequence of the layers is reversed then the analogy of case b) applies resulting in Eq. 26 and Eq. 27:

$$\begin{aligned} X &= X_1 + X_2 + X_3 = k_{22} X_1' + k_{21} X_2' + k_{13} X_3' \\ Y &= Y_1 + Y_2 + Y_3 = k_{22} Y_1' + k_{21} Y_2' + k_{13} Y_3' \\ Z &= Z_1 + Z_2 + Z_3 = k_{22} Z_1' + k_{21} Z_2' + k_{13} Z_3' \end{aligned} \quad (26)$$

$k_{21} = k\,[1-\exp(-\mu_2 t_2)]$; $k_{22} = k\exp(-\mu_2 t_2)][1-\exp(-\mu_1 t_1)]$;

$k_{13} = k\exp(-\mu_1 t_1 - \mu_2 t_2)]$

$$\begin{aligned} x &= X/(k_{22}V_1 + k_{21}V_2 + k_{13}V_3) \\ y &= Y/(k_{22}V_1 + k_{21}V_2 + k_{13}V_3) \end{aligned} \quad (27)$$

where V_1, V_2, V_3 are known values.

Printing Wet Into Wet

If both colour layers are mixed see Fig. 5, the resulting coverage by colour pigment with area s_1 and with area s_2 are shown in Eq. 28.

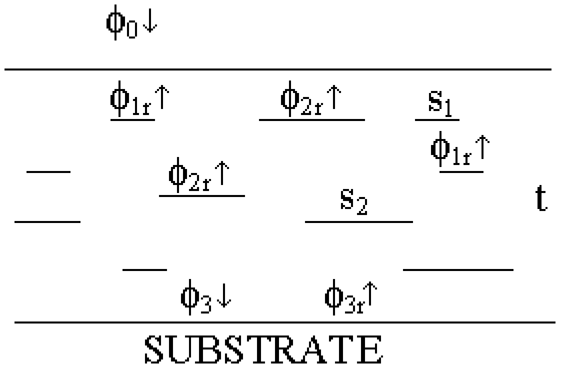


Figure 5. The blended pigment particles with areas s_1 and s_2

$$P = (P_1 - 0{,}5\, P_1 \cap P_2)_{c1} + (P_2 - 0{,}5\, P_1 \cap P_2)_{c2}$$

$$P = 0{,}5[1 - (T_1 - T_2) - T]_{c1} + 0{,}5[1 + (T_1 - T_2) - T]_{c2} \quad (28)$$

The tristimulus values are given by Eq. 29 and for the substrate are the same as in the previous section.

$$X_1 = \int \phi_0\, P\, x(\lambda)\, d\lambda = 0{,}5\, k \int P_e(\lambda)[1-(T_1-T_2)-T]\, r_1(\lambda) x(\lambda)\, d\lambda + 0{,}5\, k \int P_e(\lambda)[1 + (T_1-T_2) - T]\, r_2(\lambda) x(\lambda)\, d\lambda$$

$$X_1 = k_1 \int P_e(\lambda)\, r_1(\lambda)\, x(\lambda)\, d\lambda + k_2 \int P_e(\lambda)\, r_2(\lambda)\, x(\lambda)\, d\lambda = k_1 X_1' + k_2 X_2'$$

$$Y_1 = k_1 \int P_e(\lambda)\, r_1(\lambda)\, y(\lambda)\, d\lambda + k_2 \int P_e(\lambda)\, r_2(\lambda)\, y(\lambda)\, d\lambda = k_1 Y_1' + k_2 Y_2' \quad (29)$$

$$Z_1 = k_1 \int P_e(\lambda)\, r_1(\lambda)\, z(\lambda)\, d\lambda + k_2 \int P_e(\lambda)\, r_2(\lambda)\, z(\lambda)\, d\lambda = k_1 Z_1' + k_2 Z_2'$$

where

$$k_1 = 0{,}5\, k[1 - (T_1 - T_2) - T]$$
$$k_2 = 0{,}5\, k[1 + (T_1 - T_2) - T]$$
$$k_{13} = kT$$

Special Cases of the Substrate Reflectances

In the case of absolutely black substrate we can write $X_3' = Y_3' = Z_3' = 0$, $V_3 = 0$ and for the perfect reflector $X_3' = Y_3' = Z_3'$, $V_3 = 3X_3'$. If the colour ink layers are sufficiently thin then the substrate reflectance influences the final colour gamut.

Example

We will consider a sample in which the first layer has pigment C_1 (cyan), with CIExy co-ordinates $x_1=0.045$, $y_1=0.333$, and a transmittance $T_1=0.612$. The second layer has pigment C_2 (magenta) with co-ordinates $x_2=0.5$, $y_2=0.15$ and transmittance $T_2=0.140$. The yellow substrate has a specular reflectance with colour co-ordinates $x_3=0.41$, $y_3=0.59$. Based on these values calculation demonstrates that changes have occurred to the colour gamut see below. After the calculation we can get for the tristimulus values and colour co-ordinates

$$X_C=0.136 \quad X_M=3.333 \quad X_Y=0.69$$
$$Y_C=1 \quad Y_M=1 \quad Y_Y=1$$
$$Z_C=1.894 \quad Z_M=2.333 \quad Z_Y=0$$

$$k_{11} = 1 - T_1 = 0.388$$
$$k_{12} = T_1(1-T_2) = 0.526$$
$$k_{13} = T_1 T_2 = 0.086$$

$$V_1 = X_C + Y_C + Z_C = 3.030$$
$$V_2 = X_M + Y_M + Z_M = 6.666$$
$$V_3 = X_Y + Y_Y + Z_Y = 1.690$$

$$x = (k_{11}X_C + k_{12}X_M + k_{13}X_Y) / (k_{11}V_1 + k_{12}V_2 + k_{13}V_3) = 0.059$$
$$y = (k_{11}Y_C + k_{12}Y_M + k_{13}Y_Y / (k_{11}V_1 + k_{12}V_2 + k_{13}V_3) = 0.207$$

If the colour ink layers are in sequence: the outer layer magenta, the inner layer cyan and the substrate is the same we obtain:

$$k_{22} = T_2(1-T_1) = 0.054 \qquad k_{21} = 1 - T_1 = 0.860$$

$$x = (k_{22}X_C + k_{21}X_M + k_{13}X_Y) / (k_{22}V_1 + k_{21}V_2 + k_{13}V_3) = 0.486$$

$$y = (k_{22}Y_C + k_{21}Y_M + k_{13}Y_Y) / (k_{22}V_1 + k_{21}V_2 + k_{13}V_3) = 0.166$$

Conclusion

In this paper it has been shown that self-shielding of pigment particles significantly influences the gamut of overprinted colour layers. The effect of the sequence of the layers is non-commutative. If the layer thickness is sufficiently thin it has been shown that the colour of the substrate influences the final result. In addition it is also influenced by the nature of the reflectance namely whether it is diffuse or specular. We also derived the necessary relationships, which allow us to calculate the deviations in the colour co-ordinates which can be seen in the chromatic diagram CIExy. These deviations are given by the sequence of the printed layers, ink formulation and thickness of the colour layers as well as the colour of the substrate. By using the presented mathematical model we can describe the trapping of printing more precisely.

References

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Biography

Miroslav Dohnal graduated from the Faculty of Science of the Palacký University in Olomouc, he majored in optics and fine mechanics. He was awarded doctor's degree in science RNDr. (M.Sc) in Applied physics in 1972. He received Ph.D. in Continuum mechanics from The Institute of Thermomechanics of the Czechoslovak Academy of Science in Prague in 1986. He was qualified as an associated professor at Czech Technical University, Prague, in 2000. His prime interest is the theory of the colour spaces, colourimetry, lasers and its application in graphic arts. Since 1993 he has given lectures in these subjects at the University of Pardubice.