Printer Characterization Using Adaptive Constraint Optimization

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Abstract

In this paper we present a novel fast method to find a specific colorant value (eg. CMYK) that has to be sent to a printer in order to obtain a required color appearance. Due to the general approach, the creation of ICC-profiles is also possible for printers using more than 4 ink types (eg. CcMmYK). The proposed method minimizes a prescribed cost function that can be used for example for GCR or UCR, in the region of the valid colorant values. At first, we construct a printer model based on a set of printed color patches, using the multilinear or "Partition of Unity"-method interpolation. In the second step, the space of the colorant values is cut into parallelepipeds which have to contain the desired colorant value. The parallelepipeds are sorted with the aid of the cost function and then step by step adaptively tested for valid colorant values. On a Athlon 1,2 GHz the colorant calculation in the CMYK color space takes about 3 ms. Measurement results are given in the text.

Introduction and Task

With the availability of printers using more than 3 ink types (e.g. CMYK, CcMmYK) the construction of precise printer models and their fast inversion has become an increasing research area. Therefore a typical problem is on the one hand to find the amount of possible colorant values that reproduce a given color appearance and on the other hand to select the best of them in a specific manner (e.g. for GCR or UCR).

In recent years different models (e.g. Neugebauer [6], Yule-Nielson [10] and their derivatives) was used to characterizing printers. This models were developed for the 3 ink printing process but can be adjusted in some cases to printers using more colorants. To invert the models different methods have been developed: Cholewo [2] also used constraint optimization but solved the problem with lagrange multiplier algorithm, which is numerically difficult to handle (calculating Jacobian, positive definiteness of Hessian, etc.) and poses problems like trapping into local minimums [3]. Tominaga [8] used neural networks, but this method does not allow specific black generation. In this paper we construct two general printer models based on multilinear or "Partition of Unity"-method [1] interpolation, and a novel fast method based on adaptive constraint optimization to invert this models considering human visual thresholds and printer tolerances. The paper is organized as follows: At first we will give a mathematical formulation of the task that has to be solved. In the next section the possible printer models for our method will be described. Then we present the algorithm which is based on a theorem that we will prove before. Finally, some measurement results and conclusions will be given.

In the following we describe any printer P with the function $\mathcal{F}_P : \Omega_{ink} \longrightarrow CIELab$ (Ω_{ink} specifies the colorant values of the printer, eg. $CMYK$). The task is the calculation of a specific colorant value $\chi \in \Omega_{ink}$ that has to be sent to the printer to obtain a required color appearance $\vartheta \in CIELab$. This means that we must find the inverse function $\mathcal{F}_P^{-1} : CIELab \longrightarrow \Omega_{ink}$ of $\mathcal{F}_P$. But in general $\mathcal{F}_P$ is neither surjective ($\mathcal{F}_P(\Omega_{ink}) \neq CIELab$) nor injective ($\exists \vartheta \in CIELab, \exists \chi_1, \chi_2 \in \Omega_{ink}, \chi_1 \neq \chi_2 : \mathcal{F}_P(\chi_1) = \mathcal{F}_P(\chi_2) = \vartheta$). The inverse function $\mathcal{F}_P^{-1}$ does not exist. In the following we explore the case that any required color appearance $\vartheta \in CIELab$ is in the printer gamut $\vartheta \in \mathcal{F}_P(\Omega_{ink}) \subset CIELab$. The gamut mapping method to achieve this case is not part of the paper.

The algorithm that we explain solves the following

Task : A color appearance $\vartheta \in \mathcal{F}_P(\Omega_{ink}) \in CIELab$ and a continuous cost function $\mathcal{F}_{cost} : \Omega_{ink} \longrightarrow \mathbb{R}$ are given. We want to find a colorant value $\chi \in \Omega_{ink}^{\vartheta} := \{\omega \in \Omega_{ink} | \, \|\mathcal{F}_P(\omega) - \vartheta\|_2 < \Delta E_{visible}\}$ that minimizes the cost function $\mathcal{F}_{cost}(\chi) = \min_{\psi \in \Omega_{ink}^{\vartheta}} \mathcal{F}_{cost}(\psi)$.

1. $\mathcal{F}_{cost}$ can be used for GCR or UCR.
2. $\Delta E_{visible}$ is the human visual threshold where color differences are not recognized below.

Printer Model

Since generally $\mathcal{F}_P$ is not known, we construct a printer model based on a set of printed color patches with known colorant values $\Omega_{ink}^{test} \in \Omega_{ink}$ (eg. IT8.7/3 for $\Omega_{ink} \in CMYK$), using the multilinear or "partition of unity"-method interpolation. $\mathcal{F}_P$ will be replaced below by the interpolation function $\mathcal{F}_P^{\Omega_{ink}^{test}} : \Omega_{ink} \longrightarrow CIELab$.

Definitions and Terminology

Definition 1 We define the multiplication of two vectors $\xi \in \{0,1\}^n, a \in \mathbb{R}^n$ by $\xi * a := (\xi_1 a_1, \ldots, \xi_n a_n)^T \in \mathbb{R}^n$.

Definition 2 Let $a, b \in \mathbb{R}^n$, $a < b :\Longleftrightarrow a_i < b_i$, $i = 1, \ldots, n$. Then we denote with ${}_a\Box^b := \{\xi \in \mathbb{R}^n \mid a_i \leq \xi_i \leq b_i,\ i = 1, \ldots, n\}$ a parallelepiped in $\mathbb{R}^n$, that is spanned by a and b. $E({}_a\Box^b) := \{a + I * (b - a) \mid I \in \{0,1\}^n\}$ is the set of 2^n corner points of ${}_a\Box^b$.

Definition 3 For $I = (\lambda_1, \ldots, \lambda_n)^T \in \{0,1\}^n$ we define:

1. $I_j := \lambda_j$
2. $\overline{k} := \begin{cases} 0 & \text{for} \quad k = 1 \\ 1 & \text{for} \quad k = 0 \end{cases}, k \in \{0,1\}$
3. $I^j := (\lambda_1, \ldots, \lambda_{j-1}, \overline{\lambda_j}, \lambda_{j+1}, \ldots, \lambda_n)^T \in \{0,1\}^n$
4. $\overline{I} := (\overline{\lambda_1}, \ldots, \overline{\lambda_n})^T \in \{0,1\}^n$

Definition 4 We subdivide a not degenerated parallelepiped ${}_a\Box^b \subset \mathbb{R}^n$, $a, b \in \mathbb{R}^n$, into 2^n parallelepipeds which have equivalent volume. The set of parallelepipeds ${}_a\boxplus^b$ can be written as follows

$$ {}_a\boxplus^b \quad := \quad \left\{ {}_{a+\frac{1}{2}(I*(b-a))}\Box^{b-\frac{1}{2}(\overline{I}*(b-a))} \mid \forall I \in \{0,1\}^n \right\}. $$

Definition 5 Let $P := \{p_1, \ldots, p_k\} \subset \mathbb{R}^n$ be a point set and $\mathcal{F} : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ a function. Then we denote with $\mathcal{C}_{\mathcal{F}}(P)$ the convex hull of the mapped point set P with respect to $\mathcal{F}$. That means

$$ \mathcal{C}_{\mathcal{F}}(P) := \left\{ \xi \in \mathbb{R}^m \mid \xi = \sum_{i=1}^{k} \lambda_i \mathcal{F}(p_i),\ \lambda_i \in [0,1],\ \sum_{i=1}^{k} \lambda_i = 1 \right\}. $$

Definition 6 With $\mathcal{Q}_{\mathcal{F}}(P) \in \mathbb{R}^n$ we denote the smallest parallelepiped ${}_a\Box^b \in \mathbb{R}^n$ that contains the convex hull of the mapped point set P with respect to $\mathcal{F}$. That means

$$ \mathcal{Q}_{\mathcal{F}}(P) \quad := \quad \min_{a,b \in \mathbb{R}^n} \left\{ {}_a\Box^b \mid \mathcal{C}_{\mathcal{F}}(P) \subset {}_a\Box^b \right\}. $$

Definition 7 With the continuous function $\mathcal{F} : \mathbb{R}^n \longrightarrow \mathbb{R}$ we define a reflexive quasiorder $\leq_{\mathcal{F}}^{\Box}$ on the finite set of parallelepipeds $\mathcal{M} = \{{}_{a_i}\Box^{b_i} \subset \mathbb{R}^n \mid i = 1, \ldots, m\}$ as follows

$$ \Box_1, \Box_2 \in \mathcal{M} : \Box_1 \leq_{\mathcal{F}}^{\Box} \Box_2 :\Longleftrightarrow \min_{\xi \in \Box_1} \mathcal{F}(\xi) \leq \min_{\xi \in \Box_2} \mathcal{F}(\xi). $$

Definition 8 Let $\mathcal{R}$ be an orthogonal grid, that is spanned over $\Omega = {}_a\Box^b \subset \mathbb{R}^n$. We say $\mathcal{R}$ has an open disjunct mesh set $\mathcal{G}_{\Omega}^{\mathcal{R}} := \{{}_{a_i}\Box^{b_i} \subset \Omega \mid E({}_{a_i}\Box^{b_i}) \subset \mathcal{R}, i = 1, \ldots, m\}$, if

$$ \begin{aligned} \bigcup_{{}_a\Box^b \in \mathcal{G}_{\Omega}^{\mathcal{R}}} {}_a\Box^b &= \Omega \\ \bigcap_{{}_a\Box^b \in \mathcal{G}_{\Omega}^{\mathcal{R}}} \overset{\circ}{{}_a\Box^b} &= \emptyset, \text{ with } \overset{\circ}{{}_a\Box^b} := \{\xi \in \Omega \mid a_i < \xi_i < b_i\}. \end{aligned} $$

Multilinear Interpolation

We span an orthogonal grid on Ω_{ink} with nodal points in Ω_{ink}^{test}. Every mesh $Q_m \subset \Omega_{ink}$ is a closed parallelepiped with dimension $\mathrm{Dim}(\Omega_{ink})$ and corner points $c_I^m \in \Omega_{ink}^{test}$, $I \in \{0,1\}^{\mathrm{Dim}(\Omega_{ink})}$. The corner points are ordered in that way, that they fulfill the condition (reminding Definition 3): $c_I^m - c_{I^j}^m = (0, \ldots, 0, \underset{\substack{\downarrow \\ \text{position } j}}{\left\| c_I^m - c_{I^j}^m \right\|_2}, 0, \ldots, 0)^T$. The interpolation function will be defined for every point $\chi \in Q_m$ as follows

$$ \begin{aligned} \mathcal{F}_P^{\Omega_{ink}^{test}}(\chi) \quad = \quad & \sum_{I \in \{0,1\}^{\mathrm{Dim}(\Omega_{ink})}} \left(\prod_{j=1}^{\mathrm{Dim}(\Omega_{ink})} \frac{|(c_I^m)_j - \chi_j|^{I_j}}{\left\| c_I^m - c_{I^j}^m \right\|_2} \right. \\ & \left. (1 - \frac{|(c_I^m)_j - \chi_j|}{\left\| c_I^m - c_{I^j}^m \right\|_2})^{1-I_j} \right) \mathcal{F}_P(c_I^m) \end{aligned} \tag{1} $$

It can be seen that the interpolation function $\mathcal{F}_P^{\Omega_{ink}^{test}} \in C^0(\Omega_{ink}, CIELab)$ is only continuous.
If $\mathrm{Dim}(\Omega_{ink}) = 3$ this interpolation is the well known trilinear interpolation.

"Partition of Unity"-Method Interpolation

With this method the interpolation function can be n-times continuously differentiable by construction, $n \in \mathbb{N}_0 \cup \{\infty\}$. Below we denote with $N = \#(\Omega_{ink}^{test})$ the number of test color patches and $\{\xi_1, \ldots, \xi_N\} = \Omega_{ink}^{test}$. We construct an open cover $\Psi_m \subset \Omega_{ink}$, $m = 1, \ldots, N$ of Ω_{ink} with the properties

$$ \begin{aligned} \bigcup_{m=1}^{N} \Psi_m &= \Omega_{ink} \\ \forall \Psi_m, \exists \xi_m \in \Omega_{ink}^{test} &: \xi_m \in \Psi_m \\ \Psi_m \neq \Psi_n, \nexists \xi \in \Omega_{ink}^{test} &: \xi \in \Psi_m \cap \Psi_n \end{aligned} $$

Additionally we define functions $\varphi_m \in C^n(\Omega_{ink}, [0,1])$, $m = 1, \ldots, N$, $n \in \{1, \ldots, \infty\}$ with compact support as follows

$$ \varphi_m(\chi) = \begin{cases} 1 & , \quad \chi \in \Psi_m \cap \Omega_{ink}^{test} \\ \neq 0 & , \quad \chi \in \Psi_m \\ 0 & , \quad \chi \notin \Psi_m \end{cases} $$

Then we can define an n-times continuously differentiable interpolation function

$$ \mathcal{F}_P^{\Omega_{ink}^{test}}(\chi) = \sum_{m=1}^{N} \frac{\varphi_m(\chi)}{\sum_{i=1}^{N} \varphi_i(\chi)} \mathcal{F}_P(\xi_m) \tag{2} $$

Remarks

1. $\mathcal{F}_P^{\Omega_{ink}^{test}}$ fulfils the data consistency condition, i.e. $\forall \xi_k \in \Omega_{ink}^{test}$:

$$ \begin{aligned} \mathcal{F}_P^{\Omega_{ink}^{test}}(\xi_k) \quad &= \quad \sum_{m=1}^{N} \frac{\overbrace{\varphi_m(\xi_k)}^{=1 \text{ for } m=k, \text{ otherwise } 0}}{\sum_{i=1}^{N} \underbrace{\varphi_i(\xi_k)}_{=1 \text{ for } i=k, \text{ otherwise } 0}} \mathcal{F}_P(\xi_m) \\ &= \quad \mathcal{F}_P(\xi_k) \end{aligned} \tag{3} $$

2. If we choose the test color patches Ω_{ink}^{test} forming an equidistant orthogonal grid, (nodal point distance is q) then we can choose Ψ_m as an open sphere $\Psi_m = \{\chi \in \Omega_{ink} \mid \|\xi_m - \chi\|_2 < q\}$. Now we can define the φ_m as radial symmetric functions, e.g as C^∞-functions

$$\varphi_m = \begin{cases} \Omega_{ink} & \longrightarrow & [0,1] \\ \chi & \rightarrow & \begin{cases} e^{-\frac{\|\xi_m-\chi\|_2^2}{q^2-\|\xi_m-\chi\|_2^2}} & ,\chi \in \Psi_m \\ 0 & ,\text{otherwise} \end{cases} \end{cases}$$

In this case $\mathcal{F}_P^{\Omega_{ink}^{test}}$ is also a C^∞-function which is easy to evaluate, because in every mesh the sum (2) resolves into maximal $2^{\mathrm{Dim}(\Omega_{ink})}$ addends.

Algorithm

Mathematical Background
Now we have a closer look to the multilinear interpolation (equation 1) and to the interpolation using "Partition of unity"-method (equation 2) , considering the remarks given above. The weighting factors $(\lambda_1, \ldots, \lambda_n) \in \mathbb{R}^n$ in every grid mesh ${}_{a_i}\Box^{b_i}$ fulfill the following conditions: $\lambda_j \in [0,1]$, $j = 1, \ldots, n$, $\sum_{j=1}^n \lambda_j = 1$. Therefore both interpolation methods are convex combinations of the function values of the mesh corner points. That means

$$\mathcal{F}_P^{\Omega_{ink}^{test}}({}_{a_i}\Box^{b_i}) \subset \mathcal{C}_{\mathcal{F}_P^{\Omega_{ink}^{test}}}(E({}_{a_i}\Box^{b_i})) \subset \mathcal{Q}_{\mathcal{F}_P^{\Omega_{ink}^{test}}}(E({}_{a_i}\Box^{b_i})).$$

The outcome of this is the following theorem about the existence of a solution $\chi \in \Omega_{ink}$.

Theorem 1 Let $\mathcal{F}_P^{\Omega_{ink}^{test}} : \Omega_{ink} \longrightarrow CIELab$ be an interpolation function of the above form on an orthogonal grid with an open disjunct mesh set $\mathcal{G}_{\Omega_{ink}}^{\Omega_{ink}^{test}} := \{{}_{a_i}\Box^{b_i} \subset \Omega_{ink} \mid i = 1, \ldots, m\}$ and $\vartheta \in \mathcal{F}_P^{\Omega_{ink}^{test}}(\Omega_{ink})$ arbitrarily given. If $\Lambda^\vartheta := \left\{ {}_a\Box^b \in \mathcal{G}_{\Omega_{ink}}^{\Omega_{ink}^{test}} \mid \vartheta \in \mathcal{Q}_{\mathcal{F}_P^{\Omega_{ink}^{test}}}(E({}_a\Box^b)) \right\}$ then we gain

1. Necessary condition
   If there exists $\chi \in \Omega_{ink}$ with $\mathcal{F}_P^{\Omega_{ink}^{test}}(\chi) = \vartheta$ then
   $$\exists {}_a\Box^b \in \Lambda^\vartheta : \chi \in {}_a\Box^b \tag{4}$$
2. Sufficient condition
   $$\Lambda^\vartheta \neq \emptyset \Rightarrow \exists \chi \in \bigcup_{{}_a\Box^b \in \Lambda^\vartheta} {}_a\Box^b : \mathcal{F}_P^{\Omega_{ink}^{test}}(\chi) = \vartheta \tag{5}$$

Proof: First we proof the sufficient condition: Since $\mathcal{G}_{\Omega_{ink}}^{\Omega_{ink}^{test}}$ is open disjunct and $\vartheta \in \mathcal{F}_P^{\Omega_{ink}^{test}}(\Omega_{ink})$ is given, there exists a mesh ${}_a\Box^b \in \mathcal{G}_{\Omega_{ink}}^{\Omega_{ink}^{test}}$ with $\vartheta \in \mathcal{F}_P^{\Omega_{ink}^{test}}({}_a\Box^b)$. This means $\exists \chi \in {}_a\Box^b : \mathcal{F}_P^{\Omega_{ink}^{test}}(\chi) = \vartheta$. Since we permit only interpolation functions of the above form, it follows $\mathcal{F}_P^{\Omega_{ink}^{test}}({}_a\Box^b) \subset \mathcal{C}_{\mathcal{F}_P^{\Omega_{ink}^{test}}}(E({}_a\Box^b)) \subset \mathcal{Q}_{\mathcal{F}_P^{\Omega_{ink}^{test}}}(E({}_a\Box^b)) \Rightarrow {}_a\Box^b \in \Lambda^\vartheta \Rightarrow \chi \in \bigcup_{{}_a\Box^b \in \Lambda^\vartheta} {}_a\Box^b$.

Now we step to the necessary condition: If $\mathcal{F}_P^{\Omega_{ink}^{test}}(\chi) = \vartheta$, $\chi \in \Omega_{ink}$ then $\chi \in {}_a\Box^b \in \mathcal{G}_{\Omega_{ink}}^{\Omega_{ink}^{test}}$ ($\mathcal{G}_{\Omega_{ink}}^{\Omega_{ink}^{test}}$ is open disjunct) and $\vartheta \in \mathcal{F}_P^{\Omega_{ink}^{test}}({}_a\Box^b) \subset \mathcal{Q}_{\mathcal{F}_P^{\Omega_{ink}^{test}}}(E({}_a\Box^b))$, so ${}_a\Box^b \in \Lambda^\vartheta \neq \emptyset$. □

Thus with Λ^ϑ we get all meshes with a possible solution of the problem. Such we can reduce our search area in the colorant space Ω_{ink} significantly. To get Λ^ϑ rapidly the algorithm deals with look-up tables, which are required to get the mesh set Λ^ϑ for a special subspace of the $CIELab$-space. On Λ^ϑ the research for the optimal colorant value can be continued.
The algorithm to solve the task will be decomposed into the following parts

1. Constructing of look-up tables (once)
2. Calculation of the wanted colorant value for a given color appearance (figure 1)
   (a) Reduction of the search area
   (b) Adaptive mesh search

Constructing Look-Up Tables
1. We divide Ω_{ink} into an open disjunct mesh set (Definition 8) $\mathcal{G}_{\Omega_{ink}}^{\Omega_{ink}^{test}} := \{{}_{a_i}\Box^{b_i} \subset \Omega_{ink} \mid i = 1, \ldots, m\}$ based on an orthogonal grid with the reference color patches as nodal points.
2. We also divide $\mathcal{F}_P^{\Omega_{ink}^{test}}(\Omega_{ink}) \subset CIELab$ into an open disjunct mesh set $\mathcal{G}_{\mathcal{Q}_{\mathcal{F}_P}(\Omega_{ink}^{test})}^k := \{{}_{a_i}\Box^{b_i} \subset \mathcal{Q}_{\mathcal{F}_P}(\Omega_{ink}^{test}) \mid i = 1, \ldots, k\}$ based on an orthogonal grid.
3. Then we associate to every mesh ${}_a\Box_i^b \in \mathcal{G}_{\mathcal{Q}_{\mathcal{F}_P}(\Omega_{ink}^{test})}^k$, $i = 1, \ldots, k$ all meshes in ${}_\alpha\Box^\beta \in \mathcal{G}_{\Omega_{ink}}^{\Omega_{ink}^{test}}$ with

$$\mathcal{Q}_{\mathcal{F}_P^{\Omega_{ink}^{test}}}(E({}_\alpha\Box^\beta)) \cap {}_a\Box_i^b \neq \emptyset.$$

For $i = 1, \ldots, k$ we get the following sets

$$\Lambda_i := \left\{ {}_\alpha\Box^\beta \in \mathcal{G}_{\Omega_{ink}}^{\Omega_{ink}^{test}} \mid \mathcal{Q}_{\mathcal{F}_P^{\Omega_{ink}^{test}}}(E({}_\alpha\Box^\beta)) \cap {}_a\Box_i^b \neq \emptyset \right\} \tag{6}$$

Reduction of the Search Area
The colorant $\vartheta \in \mathcal{F}_P^{\Omega_{ink}^{test}}(\Omega_{ink})$ is given. Since the mesh set $\mathcal{G}_{\mathcal{Q}_{\mathcal{F}_P}(\Omega_{ink}^{test})}^k$ is open disjunct and $\vartheta \in \mathcal{F}_P^{\Omega_{ink}^{test}}(\Omega_{ink}) \subset \mathcal{Q}_{\mathcal{F}_P}(\Omega_{ink}^{test})$ there exists at least one $i \in \{1, \ldots, k\}$ with $\vartheta \in {}_a\Box_i^b \in \mathcal{G}_{\mathcal{Q}_{\mathcal{F}_P}(\Omega_{ink}^{test})}^k$ (If ϑ lies on a grid line there exist more then one i). We are searching for a mesh ${}_a\Box_i^b \in \mathcal{G}_{\mathcal{Q}_{\mathcal{F}_P}(\Omega_{ink}^{test})}^k$ that contains ϑ and look up the appropriate mesh set Λ_i in the table. According Theorem 1 (necessary condition) all meshes ${}_{\alpha_m}\Box^{\beta_m} \in \Lambda_i$ with $\vartheta \notin \mathcal{Q}_{\mathcal{F}_P^{\Omega_{ink}^{test}}}(E({}_{\alpha_m}\Box^{\beta_m}))$ do not need to be taken into account. Therefore we only investigate the following subset

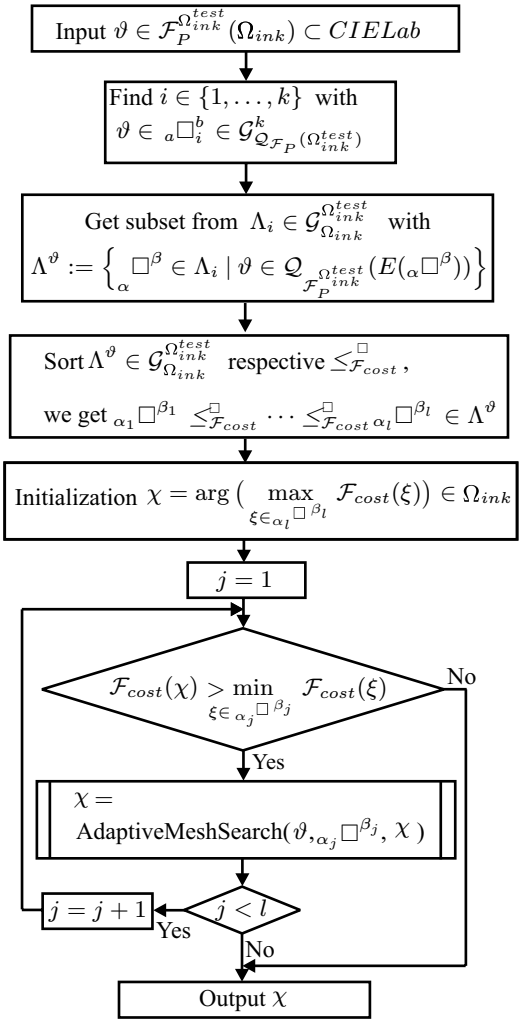

Figure 1: Flow chart of the search algorithm

$\Lambda^\vartheta := \left\{ {}_\alpha\square^\beta \in \Lambda_i \mid \vartheta \in \mathcal{Q}_{\mathcal{F}_P^{\Omega_{ink}^{test}}}(E({}_\alpha\square^\beta)) \right\}$ of Λ_i. According Theorem 1 (sufficient condition) and the precondition $\vartheta \in \mathcal{F}_P^{\Omega_{ink}^{test}}(\Omega_{ink})$ we know that there is at least one mesh in Λ^ϑ which contains the wanted colorant value $\chi \in \Omega_{ink}$. To accelerate the search, we have to assort Λ^ϑ with respect to the reflexive quasiorder $\leq^\square_{\mathcal{F}_{cost}}$. If we investigate this series of meshes (adaptive mesh search) in an ascending order, we can stop after finding a colorant value $\chi \in {}_{\alpha_j}\square^{\beta_j}$ with $\left\| \mathcal{F}_P^{\Omega_{ink}^{test}}(\chi) - \vartheta \right\|_2 < \Delta E_{visible}$ and $\mathcal{F}_{cost}(\chi) \leq \min_{\xi \in {}_{\alpha_{j+1}}\square^{\beta_{j+1}}} \mathcal{F}_{cost}(\xi)$, since in the other meshes ${}_{\alpha_{j+1}}\square^{\beta_{j+1}}, \ldots, {}_{\alpha_l}\square^{\beta_l}$ the cost function values cannot be diminished. χ is then the desired colorant value.

Adaptive Mesh Search

The search is an adaptive recursive process, in which the mesh ${}_\alpha\square^\beta \subset \Omega_{ink}$, that forms a parallelepiped is separated into sub-parallelepipeds. These sub-parallelepipeds are tested for validity and, if necessary, separated again or discarded. If no further improvement of the cost function

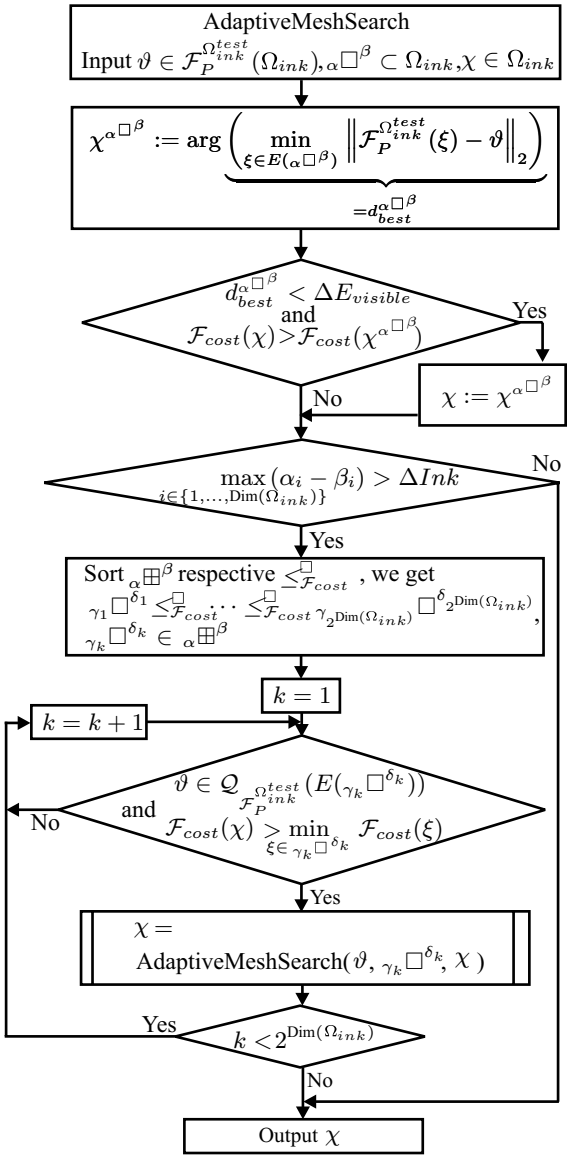

Figure 2: Flow chart of the adaptive mesh search

values is possible, the process has to be terminated . In the following we describe a recursion step for the parallelepiped ${}_\alpha\square^\beta \subset \Omega_{ink}$ (figure 2). Here $\chi \in \Omega_{ink}$ is a solution in a previous recursion step.

At first we look for the corner point $\chi^{{}_\alpha\square^\beta} \in E({}_\alpha\square^\beta)$ with

$$\chi^{{}_\alpha\square^\beta} := \arg\left(\underbrace{\min_{\xi \in E({}_\alpha\square^\beta)} \left\| \mathcal{F}_P^{\Omega_{ink}^{test}}(\xi) - \vartheta \right\|_2}_{d_{best}^{{}_\alpha\square^\beta}} \right) \tag{7}$$

and get the minimal distance $d_{best}^{{}_\alpha\square^\beta}$. If $d_{best}^{{}_\alpha\square^\beta}$ is smaller than the visual threshold $\Delta E_{visible}$ and $\mathcal{F}_{cost}(\chi^{{}_\alpha\square^\beta}) < \mathcal{F}_{cost}(\chi)$, then a valid colorant value is found, that also minimizes the cost function. Then we set $\chi := \chi^{{}_\alpha\square^\beta}$. If the parallelepiped is already small enough, that means the biggest side $\max_{i \in \{1,\ldots,\mathrm{Dim}(\Omega_{ink})\}}(\alpha_i - \beta_i)$ falls be-

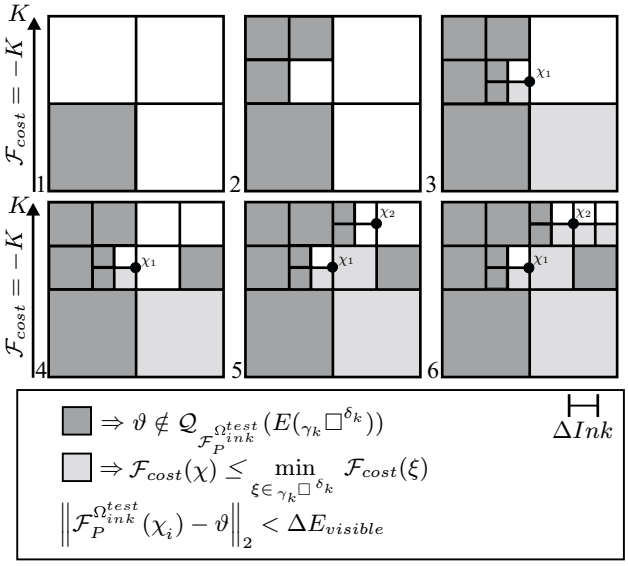

Figure 3: *Adaptive mesh search*

low a threshold ΔInk dependent on the printer, then the recursion step is terminated, because a further refinement would not lead to better results. If this is not the case, we consider the amount of sub-parallelepipeds ${}_{\alpha}\boxplus^{\beta}$ from ${}_{\alpha}\square^{\beta}$. For performance reasons we assort ${}_{\alpha}\boxplus^{\beta}$ with respect to the reflexive quasiorder $\leq^{\square}_{\mathcal{F}_{cost}}$ (Definition 7) and get an assurgent series

$$ {}_{\gamma_1}\square^{\delta_1} \leq^{\square}_{\mathcal{F}_{cost}} \cdots \leq^{\square}_{\mathcal{F}_{cost}} {}_{\gamma_{2^{\mathrm{Dim}(\Omega_{ink})}}}\square^{\delta_{2^{\mathrm{Dim}(\Omega_{ink})}}} \quad (8) $$

of sub-parallelepipeds ${}_{\gamma_k}\square^{\delta_k} \in {}_{\alpha}\boxplus^{\beta}$. In order to reduce the search-space, we only need to examine the sub-parallelepipeds ${}_{\gamma_k}\square^{\delta_k} \in {}_{\alpha}\boxplus^{\beta}$ (with the shown here recursion) for which the following both conditions are valid

$$ \vartheta \in \mathcal{Q}_{\mathcal{F}_P^{\Omega_{ink}^{test}}}(E({}_{\gamma_k}\square^{\delta_k})) \quad (9) $$

$$ x^{best}_{\mathcal{F}_{cost}} > \min_{\xi \in {}_{\gamma_k}\square^{\delta_k}} \mathcal{F}_{cost}(\xi). \quad (10) $$

Results

We have reviewed the algorithm on a HP-DesignJet 1055 CM $CMYK$-printer printing at first 6561 test color patches on normal and on coated paper. We have selected the set of color patches as follows

$$ \Omega_{ink}^{test} := \{0, 10, 20, 30, 40, 50, 60, 80, 100\}^4 \subset CMYK. \quad (11) $$

To calculate the printer model we have chosen the multilinear interpolation (equation 1). Measurements of color patches have been made with an X-Rite DTP41 spectrophotometer, which used D50 illuminant, a 0°/45° measuring geometry and the 2° standard observer. We have chosen the color appearances $\vartheta_i \in \mathcal{F}_P^{\Omega_{ink}^{test}}(\Omega_{ink}) \subset CIELab$ which have to be reproduced, that way that they are uniformly distributed to the whole printer gamut. Therefore we have divided each coordinate of the $CIELab$ space as

Table 1: **Results for normal paper**

	Min K	Max K
# of color patches in Ω_{ink}^{test}	6561	6561
# of tested color appearances	5062	5062
Mean error ΔE	1.67	1.59
Standard deviation of error ΔE	0.85	0.89
Mean black ink coverage	1.9%	18.0%
Mean calculation time for a colorant value (Athlon 1.2 GHz)	3.4ms	3.6ms

follows

$$ L_{test} = \left\{ i \cdot \frac{100}{51} \mid i = 0, \ldots, 51 \right\} $$

$$ a_{test} = b_{test} = \left\{ -128 + i \cdot \frac{255}{51} \mid i = 0, \ldots, 51 \right\} $$

and have selected the reproducible color appearances from the set

$$ T_{val} := \mathcal{F}_P^{\Omega_{ink}^{test}}(\Omega_{ink}) \bigcap (L_{test} \times a_{test} \times b_{test}) . \quad (12) $$

For the normal paper we have had to review 5062 and for the coated paper 7288 received color appearances. For both paper types we have checked the algorithm using two different cost functions

$$ \mathcal{F}^1_{cost}(C, M, Y, K) = K \quad (13) $$

$$ \mathcal{F}^2_{cost}(C, M, Y, K) = -K \quad (14) $$

Function (13) minimizes, function (14) maximizes the black ink coverage. The required thresholds we have chosen as follows

$$ \Delta E_{visible} = 1 \quad (15) $$

$$ \Delta Ink = 0.625 \quad (16) $$

The test of both paper types has been executed as follows

1. Calculating the colorant values for the color appearances from T_{val} using both cost functions (13, 14).
2. Printing the resulted colorant values.
3. Measuring the printed color patches using the spectrophotometer.
4. Comparing the measured color appearances with the wanted color appearances from T_{val}.

The results are listed in tables 1 and 2. Error curves and mean black ink coverage curves for both paper types can be seen in figure 4.

Conclusions

We have presented a fast method to invert the two printer models that have been developed. Due to the general approach printers using more than 4 ink types can be characterized. The method has been reviewed on a $CMYK$-printer using the multilinear printer model and two paper types each with two different cost functions. The test has achieved good error rates of mean error $\Delta E < 2$. In future

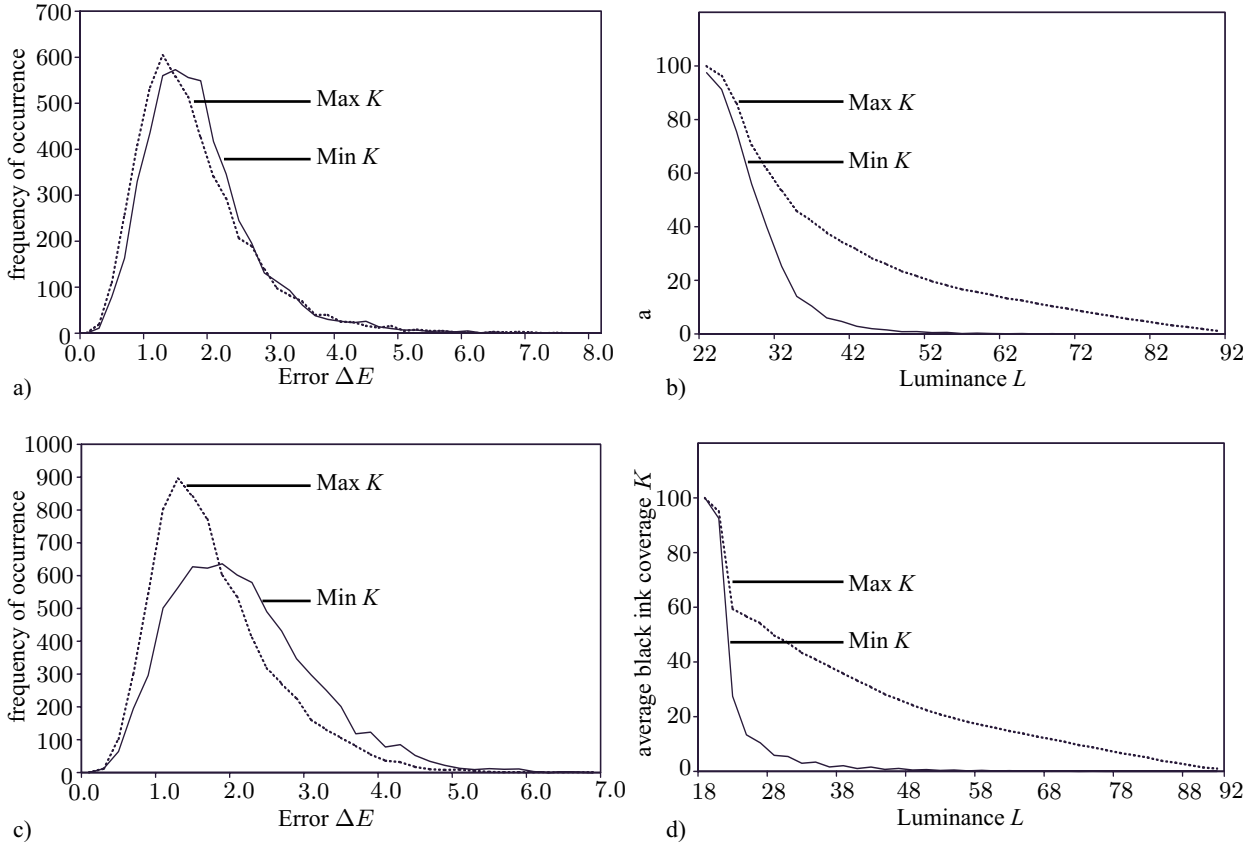

Figure 4: a) *Frequency of occurrence error function for normal paper* b) *average black ink coverage function for normal paper* c) *frequency of occurrence error function for coated paper* d) *average black ink coverage function for coated paper*

Table 2: **Results for coated paper**

	Min K	Max K
# of color patches in Ω_{ink}^{test}	6561	6561
# of tested color appearances	7288	7288
Mean error ΔE	1.97	1.60
Standard deviation of error ΔE	0.94	0.81
Mean black ink coverage	1.0%	21.0%
Mean calculation time for a colorant value (Athlon 1.2 GHz)	3.2ms	3.3ms

work the printer models and the inversion method must be reviewed on printers using 6 colorants respective the influence of the human visual threshold and printer tolerances.

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Biography

Philipp Urban was born in 1975 in Wroclaw, Poland. He received the dipl. math at the University of Hamburg, Germany in 1999. Since 1999 he is part of the research group "Vision Systems" of the Technical University Hamburg-Harburg, Germany, and works for "Ratio Entwicklungen GmbH", ICC-member. His research interests are color management, gamut boundary calculation, device calibration and characterization.