

Input Imaging Media And Their Theoretical Colour Gamuts

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Abstract

The colour gamuts of colour reproduction media are important properties of them and can play a decisive role in their use in colour reproduction applications as well as the improvement of their capabilities. While this topic has frequently been studied and is well understood and for output colour imaging media, a solution for input media is not to be had in a simple way. To this end, the present paper proposes a method for obtaining the theoretical gamut across which a device can capture colour differences and this method is based on simulating the responses of an input medium to given spectral power distributions. The gamut of an input medium is then determined on the basis of having a set of spectra that cover the majority of all possible spectra, knowing a medium's responses to them and then determining a boundary beyond which the medium does not produce variation in its responses. The present paper is an abridged version of a more extensive treatment of the topic submitted for journal publication.¹

Introduction

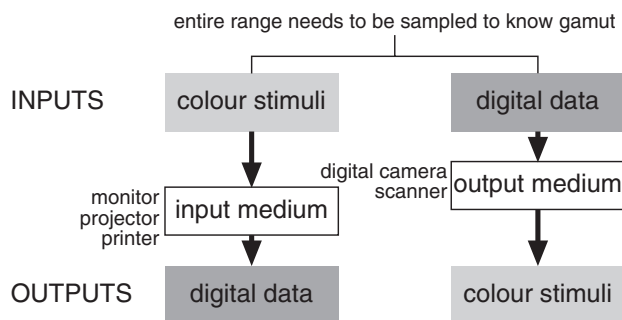


Fig. 1. Types of digital colour imaging media.

A digital colour imaging medium provides a link between digital data and colour stimuli and can be of two principal types depending on whether colour stimuli are its inputs or outputs. Output colour imaging media (e.g. monitors, printers, projectors) produce colour stimuli from digital data sent to them whereas input media (e.g. digital cameras, scanners) produce digital data by sensing colour stimuli (Fig. 1.).

Hence, what a colour imaging medium's gamut is depends on which of these classes it belongs to.² For output media the gamut is the range of colour stimuli they can produce and for input media it is the range of colour stimuli across which they can sense differences. In both cases, determining the gamut of a medium requires having

access to the entire range of inputs to the medium – for output media this means access to the entire range of digital data that can be input to them and for input media one needs access to the entire range of colour stimuli that can be presented to them for capture. Once one has access to the entire range of inputs, the gamut is then determined on the basis of a medium's corresponding outputs. Note also that it is only meaningful to determine a medium's gamut in the space of colour stimuli as in the space of digital data it is always a cube (or hypercube) or some trivial subset of it (e.g. in printers due to setting a maximum total ink amount).

Hence calculating the colour gamut of an output medium consists of sending every possible digital input (or a sample thereof) to it, measuring the colour of each corresponding output and then calculating a boundary enclosing these colours in a colour space. The generation of inputs to these media is a trivial matter as one has access to their entirety. Furthermore if the relationship between the medium's inputs and the colours of resulting stimuli (e.g. between RGB and CIELAB for a CRT) is monotonic it is sufficient to sample the extremes of the range of digital inputs (e.g. the faces of the RGB cube for a CRT) as these will correspond to the extremes of the resulting stimuli's colours – the gamut boundary. The boundary enclosing the medium's extremes can then be described using a range of methods and data structures.³⁻⁵

The reason why complexity arises for the gamuts of input media is that sampling the entire range of possible inputs to them means sampling the entire range of possible colour stimuli. This is the case because, to determine the range across which differences in stimuli can be sensed, a set of stimuli with a gamut greater than or equal to the gamut of the given input medium needs to be available. Practically this also means that the gamuts of input imaging media can only be determined theoretically as having such a set of actual surfaces is extremely difficult to achieve.

Once a set of computationally generated samples from the entire possible gamut of stimuli is available, it is necessary to know the medium's responses to each of them and then based on this data to determine the medium's gamut boundaries. The approach suggested in this paper is based on simulation whereby samples will be generated numerically in terms of their spectra and a medium's responses to them will be simulated computationally as well. Hence, the present paper will consist of three principal parts: generation of a set of stimuli for determining the gamut boundaries of input media, modelling of the responses of input media and calculation of gamut boundaries of input media.

Sampling the set of all possible stimuli

The set of all possible physical stimuli resulting in visual responses is virtually unbounded* and a sampling of its entirety is therefore virtually impossible. However, the set of all possible surface characteristics resulting in visual responses is bounded as it is determined by spectral reflection or transmission properties which are bounded themselves. While this set (referred to as the object colour solid – OCS) does not represent all possible stimuli, it does represent all possible stimuli resulting from a particular light source's output being reflected from or transmitted by all possible surfaces. The suitability of such a set for calculating gamuts of input media is further supported by these having finite range and hence there is such a theoretical stimulus spectral power distribution (SPD) beyond which increases in energy will not result in increases in sensed data – this SPD can be referred to as the maximal stimulus (S_{\max}). The necessary existence of S_{\max} (given limited range) also means that all other stimuli can be normalised by it to give relative spectra with the same characteristics as reflectance or transmission spectra (i.e. having values across the spectrum only from the interval $[0,1]^{\dagger}$). Given this constraint, the set of all possible stimuli for an input medium is the OCS. An alternative interpretation of the use of the OCS for determining an input medium's gamut is that it only represents the intersection of the medium's gamut and the gamut of all possible surfaces under a given illuminant.

The next point to consider is how best to sample the OCS. The simplest option is to generate spectral reflectances by varying reflectance values independently at a number of intervals across the visible spectrum. However, such a sampling results in samples being unevenly distributed in terms of colour appearance and to achieve a sufficiently dense sampling throughout colour space would require the calculation of an extreme amount of spectra. Using 31 intervals between 400 and 700 nm would result in 2^{31} samples if only the 0% and 100% levels are considered. Such a sampling, however, does not provide a sufficiently dense sampling in visual terms. Wanting to increase the number of reflectance levels sampled even to three would increase the number of samples to 3^{31} which is unacceptably high in terms of computation and still insufficient for sampling colour appearance space sufficiently.

A much more efficient approach is to sample in terms of a colour appearance space and then calculate spectra for each of the sampled colour space coordinates. To do this, the gamut boundary of the OCS is first needed in colour appearance space terms. In the present study CIELAB⁶ was chosen as the colour appearance space a D53 light source was used and gamuts were determined by calculating 16×16 element gamut boundary descriptors (GBDs) using the segment maxima method.⁵ To calculate the gamut boundary of the OCS, a three stage process was carried out:

- 1) Calculate the GBD for the 2^{31} reflectance spectra obtained by all combinations of the 0% and 100% levels for 31 intervals of the visible spectrum between 400 and 700 nm. This is a first approximation of the OCS's gamut and will be referred to as GBD'.
- 2) Generate further stimuli from the OCS by taking the colours in GBD' and scaling their tristimulus values to a number of levels p between 0% and 100%. The resulting XYZ values are all from the OCS as they correspond to the XYZ values of scaled versions of the spectra from GBD' and as any spectrum from GBD' scaled by p is in the OCS. Scaled versions of the spectra in GBD' can be had by scaling their XYZs as $X = k \sum S(\lambda)R(\lambda)\bar{x}(\lambda)$ (where $S(\lambda)$ is the SPD of a light source, $R(\lambda)$ the reflectance spectrum of a surface, $\bar{x}(\lambda)$ is a colour matching function and each of these is an $n \times 1$ matrix) and therefore pX can be obtained either by $pX = pk \sum S(\lambda)R(\lambda)\bar{x}(\lambda)$ or $pX = k \sum S(\lambda)pR(\lambda)\bar{x}(\lambda)$ due to the distributive nature of scalar multiplication. The same applies to Y and Z if p is the same for each of X , Y and Z .
- 3) The OCS's GBD is then calculated from the set of colours obtained in step 2.

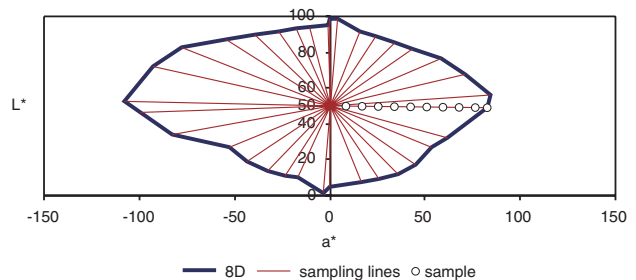


Fig. 2. Sampling gamut of colour stimuli (samples are only indicated for one sampling line).

Once the gamut boundary of the OCS is known, it can be sampled in terms of CIELAB. This sampling can, among others, be done uniformly throughout the gamut or, as will be the case here, along lines between the centre of the gamut $LAB=[50,0,0]$ and each of the 16×16 GBD colours. Why this latter strategy is used here will become clear from section 4. Note also that ten samples are taken along each such line (Fig. 2). Spectra are then generated for each of these LAB coordinates.

The method for recovering spectral reflectances corresponding to given LAB values used here is based on the concept of *metamer set* recovery.⁷ First, LAB values are transformed into CIE XYZ tristimulus values, as the relationship between these and corresponding spectra is linear. Using this linearity it is possible to recover surface reflectance for any illuminant.

Many statistical studies^{8,9} have shown that in nature spectral reflectances cover as little as 3 to 6 dimensions only, reducing the dimensionality of the colour formation problem significantly. A set of basis functions covering the most important axes of variation can be derived using characteristic vector analysis and given such a basis, reflectance is represented as a weighted average of these (a set of weights being a unique descriptor of a reflectance).

For 3 basis functions the relationship between tristimulus values and the basis weights is a linear one-to-

* It is bounded only if the amount of energy possible in this universe is bounded.

[†] This includes fluorescent samples as the S_{\max} SPD would also take into account this phenomenon.

one mapping – allowing for no metamerism (the phenomenon whereby spectrally different reflectances result in identical response). The recovery of the weights in this case is a simple matrix inverse. Three dimensions cover a lot of the OCS, however they are not sufficient for high frequency components present in saturated colours. The farther a colour is from the achromatic point (i.e. the closer it is to the OCS boundary), the higher the frequencies present in the spectrum.

To account for these high frequency components as well as to take metamerism into account, more basis functions are used – colour formation is no longer a one-to-one mapping and becomes an underdetermined system of linear equations with a possibly infinite set of solutions. The solution to such a system can be split up into a *particular* solution (from the range space of the spectral sensitivities times the illuminant) and a “black” solution (orthogonal to the range space of the spectral sensitivities times the illuminant). The former accounts for the tristimulus response and the later does not affect it and can hence be arbitrarily scaled. This results in a convex linear set, which is unbounded and represents all solutions (within the given basis).

The surface reflectances of matte, Lambertian surfaces illuminated with a diffuse light source are at any wavelength less than or equal to 100% (no more than all light is reflected) and more than or equal to 0% (no less than no light is reflected). These notions can be formulated as *linear constraints*, constituting a bounded convex set of all metamers. Given this infinite metamer set, it is possible to choose one as a representative for the set.

Of the basis dimensions tried here (ranging from 3 to 8), the 8 dimensional basis gives the best results (Fig. 3) and will therefore be used throughout the remainder of this study. Note that as the gamut of the 8D basis does not cover the entire OCS gamut, what this method will yield is the intersection of an input medium’s gamut and that of the 8D basis. While this is not an ideal solution it is significantly better than what could be done, for example, if an IT8.7/2 chart¹⁰ was used due to the 8D gamut being much larger than the gamut of the chart. This difference in gamut between that of the 8D basis and the IT8.7/2 chart can be explained by the fact that 99.8% of the variation of the chart’s spectra is covered by a 3D basis. Furthermore, the set of spectra obtained using the 8D basis could be supplemented by spectra from the surface of the OCS as these are available when the OCS gamut boundary is generated. This, however, will not be done here as it would for some sampling lines result in a large difference between the last spectrum generated using the 8D basis and the spectrum on the OCS boundary.

Even though a large proportion of the variation in natural surface spectra can be covered by three bases, it is meaningful to go to higher dimensions when determining input medium gamuts. Firstly, it can be seen from Figure 3 that the gamut of the 3D spectra does not even cover the gamut of the IT8 chart and secondly there is prior experimental evidence for

For example, Vrhel *et al.*⁸ studied 64 Munsell chips, 120 DuPont paint chips and a set of 170 reflectances of objects of varying origin (including natural and man-made samples). The approximation by a linear model of surface reflectance in the basis of principal components of the

studied surfaces was then evaluated in terms of ΔE_{ab} for an equi-energy illuminant. Importantly this study shows that large errors occur in particular for reflectances of saturated samples when using a linear model with a small number of dimensions. The analysis concludes that less than seven dimensions should suffice (although a maximum error of 5 ΔE occurs even in this seven dimensional representation). As it is the saturated colours that are of great importance in determining colour gamuts, it is meaningful to recover spectra using higher-dimensional bases.

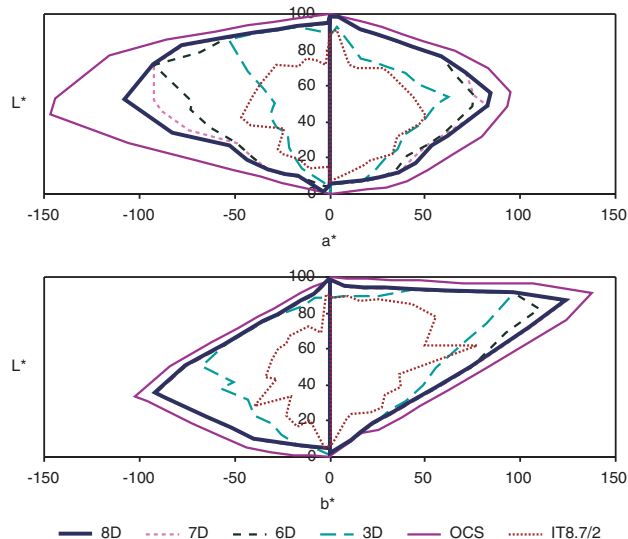


Fig. 3. Gamuts of IT8.7/3 chart, the OCS and samples obtainable using spectral bases of various dimensions.

The advantages of sampling in a colour appearance space are that from the point of view of colour appearance it is more efficient than pure spectral sampling as well as making the calculation of input medium gamuts easier, as will be shown in section 4.

Modelling the responses of input media

What digital data an input medium outputs as a result of sensing a given spectrum depends on its sensor sensitivities, dynamic response, repeatability and on the bit-depth of the digital data it outputs. Note that in the following discussion intermediate predictions of RGB values will be subscripted with the number of steps they are removed from the final prediction (i.e. R_3 is removed three steps from the final R , R_2 by two steps, etc.).

In such a model, the sensor sensitivities for a three channel input medium are represented by an $n \times 3$ matrix M (where n is the number of samples taken across the spectrum) where each column represents the sensitivities of one sensor (i.e. red, green and blue). The first approximation of the medium’s response is then obtained by:

$$[R_3 \ G_3 \ B_3] = k(R.S)^T M \quad (1)$$

where $(R.S)$ is the dot-product resulting in a $n \times 1$ column vector and $()^T$ is the transpose operator. Note that all these values are normalised by the scalar $k=1/\max(R_{3w}, G_{3w}, B_{3w})$ where R_{3w} , G_{3w} and B_{3w} are the non-normalised responses predicted for the perfect diffuser illuminated by

the chosen illuminant. Hence all $[R_3 \ G_3 \ B_3]$ values are from a range of $[0,1]$.

The $R_3G_3B_3$ responses to a given spectrum predicted by using only the spectral sensitivities of the camera assume that its response is linear and that this response varies across the entire range of input intensities. This, however, is not the case for actual media as these both have non-linearities and clipping (i.e. above and below certain input intensities, the camera's responses no longer vary) and their dynamic response therefore needs to be modelled.¹¹ These dynamic response functions (DRFs) of the medium are then represented by $m \times 2$ matrices D_R , D_G and D_B for each channel where m is the number of lightness levels at which the medium response's difference from what is predicted only using the sensor sensitivities is determined. For a given $[R_3 \ G_3 \ B_3]$ the second approximation of the response $[R_2 \ G_2 \ B_2]$ is then obtained by interpolating its values from the D_X matrices (where $X \in \{R, G, B\}$) in which the first column contains the $R_3G_3B_3$ values predicted for a series of nearly non-selective spectra (i.e. their reflectance values change little with wavelength) and the second column contains their RGB values as captured with the medium to be modelled.

Next, the effects of quantisation are modelled as follows:

$$R_1 = \text{round}(R_2/l) \quad (2)$$

Where $\text{round}()$ is a function returning the nearest integer to a given real number and l is the maximum value available at a given bit depth b given by $2^b - 1$ (e.g. 255 for 8 bits and 65535 for 16 bits). This makes the RGB values change from having a range of $[0,1]$ to having ranges that depend on bit-depth (e.g. $[0,255]$ for 8 bits per channel).

Finally a model of an input colour imaging medium also needs to acknowledge the fact that the behaviour of imaging media fluctuates and that there therefore is some repeatability error. Note that this error will not be modelled here but its extent will be determined empirically and then used in gamut calculation.

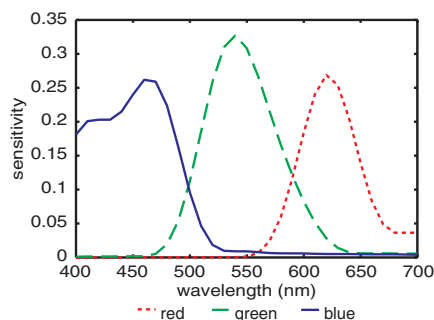


Fig. 4. Spectral sensitivities of Agfa Studiocam.

To illustrate the above input medium modelling and the subsequent gamut calculation, an Agfa Studiocam was modelled in this study. Its spectral sensitivities were recovered using the quadratic programming technique proposed by Finlayson *et al.*¹² and are shown in Fig. 4. Using this method a number of samples (i.e. the Macbeth

ColorChecker chart, 12 CERAM ceramic calibration tiles and a selection of NCS paper samples) with known spectra were captured with the camera and a constrained regression technique was then used to recover the camera's spectral sensitivities. These constraints used here included positivity (as media cannot have negative sensitivity), modality (as sensor sensitivity curves have only a small number of peaks) and band-limitedness (as sensor sensitivities are smooth).

The Studiocam's DRFs were determined for three f-stop settings (2.8, 5.6 and 11) of its lens and corresponding exposure times automatically determined by the driver software and are shown in Fig. 5.

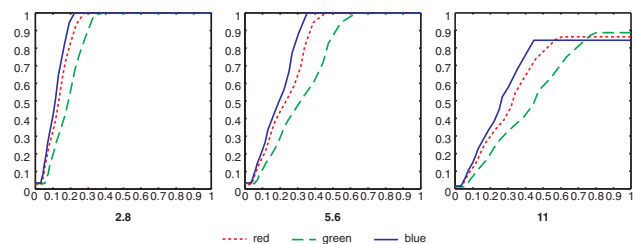


Fig. 5. Studiocam DRFs at different f-stops.

The repeatability error of the camera was 1.5% of the range in each channel and the camera was capable of quantising at 8 or 16 bits per channel.

Calculating colour gamuts of input media

The following procedure for calculating the colour gamut of an input imaging medium is directly based on the premise that the gamut of an input medium is that part of colour space across which it can sense differences and consists of the following steps:

- 1) For each of the sampling lines extending from the centre of the lightness axis towards the points of the GBD generate x CIELAB samples. (The larger x is, the more finely the gamut of a medium can be determined.)
- 2) For each LAB value from step 1 calculate the spectrum that corresponds to it given a certain illuminant and basis.
- 3) For each spectrum from step 2 calculate the responses of the camera (e.g. using the model described in section 3).
- 4) For each sampling line find that sample for which the camera response is not significantly different from the response for the following sample along that line, starting from the centre of colour space. Responses to two samples are significantly different if they differ by an amount greater than the repeatability of the camera in at least one of the channels at the given quantisation bit-depth.
- 5) The samples found in step 4 represent the gamut boundary and can be described using a GBD.

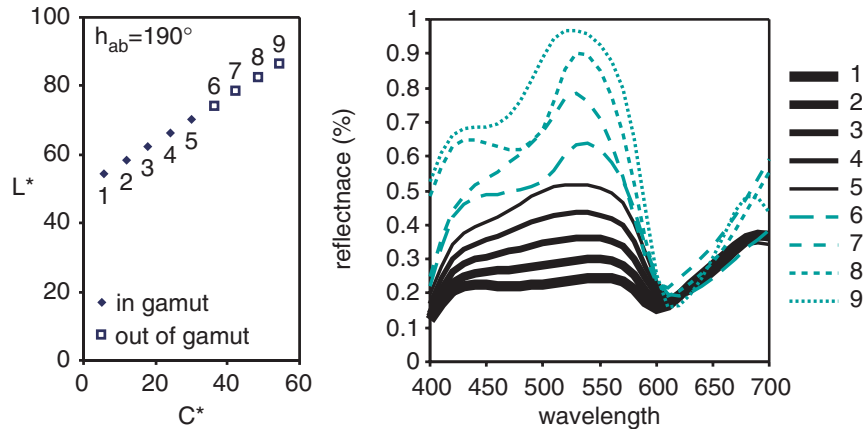


Fig. 6. Example of gamut calculation along given sampling line (f -stop = 2.8; 8 bit).

sample	1	2	3	4	5	6	7	8	9
R	0.66	0.69	0.71	0.73	0.74	0.75	0.87	0.83	0.82
G	0.69	0.84	0.98	1.00	1.00	1.00	1.00	1.00	1.00
B	0.58	0.72	0.86	0.96	1.00	1.00	1.00	1.00	1.00

Tab. 1. Camera responses to spectra shown in Fig. 6.

However, while the outcome of the above procedure is a representation of the medium's gamut boundary, it is in fact a representation that is biased. Namely, the above method can result in a gamut boundary that is smaller than the medium's actual boundary for two reasons.

Firstly, as a finite number of samples is considered along each line, the actual gamut boundary is strictly speaking in the interval determined by the sample chosen by the above method and by the next sample along each sampling line. Secondly, as a whole metamer set corresponds to each of the CIELAB values of the samples and as only one of the reflectances from the set is used in the computations for this paper, it is possible that using other reflectances that result in a given CIELAB value (i.e. that are from its metamer set) would result in a larger colour gamut being calculated.

While the first of these limitations is not one that a solution is suggested to here (beyond that of increasing the number of samples per sampling line), the second limitation can be overcome by using not only one reflectance for each CIELAB sample but by using a number of reflectances delimiting the metamer set.

To illustrate the method described above, Fig. 6 shows the LAB values and spectra along a particular sampling line and Table 1 shows the corresponding RGB responses normalised to a [0,1] range. As can be seen, the gamut boundary is between samples 5 and 6 and the fifth sample is therefore chosen to represent the boundary.

Results

The method described in section 4 has been applied to calculating the gamuts of the Agfa Studiocam for its

various states and the results of this work are presented in this section. Note that in the Figure 7 the following convention is used for labelling individual series: "Sensor, DRF_{f-stop} , $T_{\%}$, bit depth" where *Sensor* refers to the sensor sensitivities, DRF_{f-stop} is the DRF for the given f -stop setting, $T_{\%}$ is the repeatability error in terms of percentage (%) of range and *bit depth* is quantisation *bit depth*.

Fig. 7 shows the Studiocam's gamuts for the 8-bit and 1.5% repeatability case and three different f -stop settings. What can be seen here is that the choice of f -stop greatly influences the camera's colour gamut both in lightness and chroma terms.

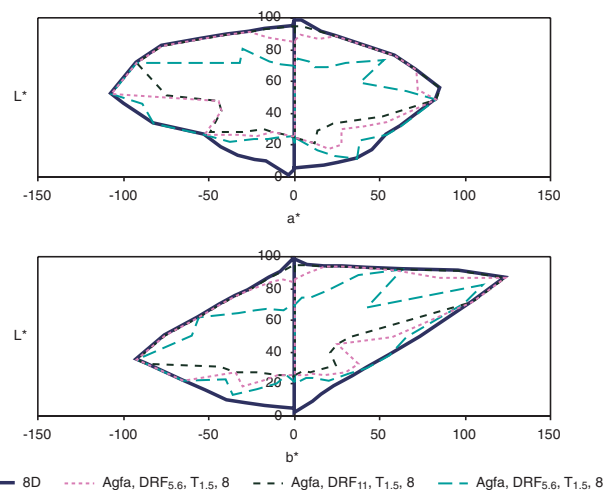


Fig. 7. Actual camera gamuts for different f -stops.

Conclusions

Overall this paper has aimed at describing methods for generating samples suitable for the calculation of the

theoretical gamuts of input media, followed by the description of a camera model and of a novel techniques for calculating input medium gamuts. In addition to being a first attempt at calculating input gamuts, the present technique also has the benefit of being of use at the stage of camera or scanner development as it could be used for tuning an input medium's parameters so as to optimise its colour gamut.

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Biography

Dr. Ján Morovic received a PhD in Colour Science at the Colour & Imaging Institute (CII) where the topic of his research was “To Develop a Universal Gamut Mapping Algorithm.” He now works at the CII as lecturer in Digital Colour Reproduction and is module leader for three modules on the MSc in Imaging Science. Further he also serves as chairman of the CIE's technical committee 8-03 on Gamut Mapping.