Influence of Noise on Color Correction

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Abstract

The transformation of device responses to CIE XYZ tristimulus values is important at an early stage in color management and this transformation is usually called color correction. It is well known that the accuracy of color correction is greatly influenced by its spectral sensitivities and noise present in the device. However, the evaluation of the influence of noise on the accuracy of color correction has not been adequately addressed.

This paper proposes the noise influence factor (NIF) to predict the influence of noise on the accuracy of color correction based on a new simple formula for a colorimetric evaluation of the device in the presence of noise. Computer simulations were performed to confirm the relationships between the NIF and the increased color correction errors by the presence of noise for each combination of sensor sets, color charts, recording and viewing illuminants and noise levels. It is shown that the proposed NIF provides a good measure to predict the influence for a variety of color sensor sets, color charts and illuminants. This predictor may be useful to design the optimal sensor sets that are robust to noise.

Introduction

At an early stage of color management, image data from a color image acquisition device are transformed to XYZ tristimulus values and this transformation is called color corrections. It is well known that the accuracy of color correction is greatly influenced by its spectral sensitivities and noise present in the device. Therefore, colorimetric evaluation of a set of color sensors is important not only to provide users with the knowledge about the colorimetric performance of the device but also to design a set of optimal spectral sensitivities. Although several colorimetric evaluation models for the device are proposed,[1-3] but they were not sufficient to predict the influence of noise on color correction. The prediction of the influence is important not only to evaluate colorimetric quality but also to evaluate the robustness of the device to noise. However it is difficult to predict the noise effect by a single measure in the previous formulations since the derived equations did not contain noise factor as a separate term.[3,4] Recently, Vora proposed a measure to evaluate the influence of noise.[5] However his noise analysis was motivated to evaluate the sensitivities of the device to noise by using the conditional number of the color correction matrix.

In this paper, a simple formula for colorimetric evaluation of a set of color sensors is derived from the different motivation to make clear the influence of noise on color correction when a signal is subject to noise.[6] This paper proposes a noise influence factor (NIF) to predict the influence of noise on the accuracy of color correction based on the proposed formula and discusses the method to estimate the noise variance of the device. From computer simulation it is shown that the proposed NIF provides a good measure to predict the influence for a variety sensor sets, object colors and recording and viewing illuminants.

Model

Colorimetric Evaluation Model

A vector space notation for color reproduction allows straight forward formulation of the problems. A sensor response vector from a set of color sensors for an object with the N x 1 spectral reflectance vector **r** can be expressed by

$$\mathbf{p} = \mathrm{S L}\mathbf{r} + \mathbf{e} \;, \tag{1}$$

where **p** is the M x 1 sensor response vector from the M channel sensors, S is the M x N matrix of the spectral sensitivities of sensors, L is the N x N diagonal matrix with samples of the spectral power distribution of a recording illuminant along the diagonal, e is the M x 1 additive noise vector and N denotes the sampling number over the visible wavelength from 400 to 700nm at 10nm intervals. For abbreviation, let denotes S_L = SL below. If the **r** and **e** are uncorrelated and the noise-mean is zero, then the recovered surface reflectance vector $\hat{\mathbf{r}}$ can be given by Wiener estimation[7],

$$\hat{\mathbf{r}} = R_{SS}S_L^T(S_L R_{SS} S_L^T + R_{ee})^{-1}\mathbf{p} \tag{2}$$

where T represents the transpose of a matrix, R_{SS} is the autocorrelation matrix of the spectral reflectance of samples which will be captured by the device and R_{ee} is also the autocorrelation matrix of the measurement noise. If noise is assumed as uncorrelated random variables, then the autocorrelation matrix can be represented by $R_{ee} = \sigma_e^2 I$, where σ_e^2 is the noise variance and I represents the identity matrix. By substituting this relation into (2), the estimation error vector $\Delta\mathbf{r}_h$ between $P_V\hat{\mathbf{r}}$ and $P_v\mathbf{r}$ is given by

$$\Delta\mathbf{r}_h = P_v\mathbf{r} - P_v R_{SS}S_L^T(S_L R_{SS} S_L^T + \sigma_e^2 I)^{-1}\mathbf{p} \;, \tag{3}$$

where P_v represents the projection matrix onto the human visual subspace (HVSS) and it is represented by

$$P_v = \sum_{i=1}^{\alpha}\mathbf{a}_i\mathbf{a}_i^T \;,$$

where $\mathbf{a}_i$ is the i-th orthonormal basis vector which spans the HVSS and

$$\alpha = Rank(TL).\ \{\mathbf{a}_i\}_{i=1-\alpha}$$

are the right singular vectors of the matrix TL, where T is the 3 x N matrix of CIE color matching functions and L is the diagonal matrix of illuminant for observation. The projected vector $P_v\mathbf{r}$ is termed a fundamental vector[8] below since the visual system is dependent only on the component of the vector that lies in the HVSS.

Since the autocorrelation matrix R_{SS} is symmetrical, it is represented by a set of eigenvectors and eigenvalues of the matrix $R_{SS} = V\Lambda V^T$, where V represents the N x N basis matrix. Λ is the N x N diagonal matrix with positive eigenvalues of the matrix along the diagonal in decreasing order. Let denotes

$$S_L^V = S_L V \Lambda^{1/2}.$$

S_L^V is termed a represented matrix of S_L by surface reflectances. Substitution of this relation into (3) leads

$$\Delta\mathbf{r}_h = P_v\mathbf{r} - P_v V\Lambda^{1/2} S_L^{V^T}(S_L^V S_L^{V^T} + \sigma_e^2 I)^{-1}\mathbf{p}. \quad (4)$$

In the above equation, the term

$$S_L^{V^T}(S_L^V S_L^{V^T} + \sigma_e^2 I)^{-1}$$

represents a simply regularized g-inverse matrix of S_L^V with respect to a regularization parameter σ_e. Let denotes the SVD of the matrix

$$S_L^V = \sum_{i=1}^{\beta} \kappa_i^v \mathbf{d}_i^v \mathbf{b}_i^{v^T},$$

where $\beta = Rank(S_L^V)$, and $\kappa_i, \mathbf{d}_i^v$ and $\mathbf{b}_i^v$

represent the i-th singular value, the i-th left and the right singular vector, respectively. Then, the simply regularized g-inverse matrix can be expressed by[9]

$$S_L^{V^T}(S_L^V S_L^{V^T} + \sigma_e^2 I)^{-1} = \sum_{i=1}^{\beta} \frac{\kappa_i^v}{\kappa_i^{v^2} + \sigma_e^2} \mathbf{b}_i^v \mathbf{d}_i^{v^T}. \quad (5)$$

Substitution of (5) into (4) and using (1) leads

$$\Delta\mathbf{r}_h = P_v\mathbf{r} - P_v V\Lambda^{1/2} \sum_{i=1}^{\beta} \frac{\kappa_i^v}{\kappa_i^{v^2} + \sigma_e^2} \mathbf{b}_i^v \mathbf{d}_i^{v^T}(S_L\mathbf{r} + \mathbf{e}). \quad (6)$$

For many surface reflectances, the 2-norm of the error vectors is averaged over the surface reflectances and the mean square error (MSE) is given by

$$E\{\|\Delta\mathbf{r}_h\|^2\} = \sum_{i=1}^{\alpha}\|\mathbf{a}_i^v\|^2 - \sum_{i=1}^{\alpha}\|P_{CV}\mathbf{a}_i^v\|^2 + \sum_{i=1}^{\alpha}\sum_{j=1}^{\beta}\left(\frac{\sigma_e^2}{\kappa_j^{v^2} + \sigma_e^2}\right)(\mathbf{b}_j^{v^T}\mathbf{a}_i^v)^2, \quad (7)$$

where P_{CV} is the projection matrix onto the subspace spanned by a set of basis vectors

$$\{\mathbf{b}_i^v\}_{i=1-\beta}$$

and is represented by

$$P_{CV} = S_L^{V^T}(S_L^V S_L^{V^T})^{-1} S_L^V = \sum_{i=1}^{\beta} \mathbf{b}_i^v \mathbf{b}_i^{v^T}. \quad (8)$$

$\mathbf{a}_i^v$ is also the represented vector of $\mathbf{a}_i$ by surface rflectances and is given by $\mathbf{a}_i^v = \Lambda^{1/2} V^T \mathbf{a}_i$. The first and the second terms in the right hand side of (7) represent the MSE for the noise free case. The value of this MSE is equivalent to that value which would be obtained by regression model since the recovered surface spectral reflectance $\hat{\mathbf{r}}$ for the noiseless case, i.e. $R_{ee} = 0$ in (2), corresponds to the recovered reflectance by regression model. Let denote this MSE MSE_{regr}. The third term corresponds to the increased MSE by the presence of noise. The left side of (7) represents the MSE by Wiener estimation and let denotes this MSE $MSE_{wien}(\sigma_e^2)$ below. Therefore, we define the third term as the noise influence factor (NIF) ant it is denoted by

$$NIF = \sum_{i=1}^{\alpha}\sum_{j=1}^{\beta}\left(\frac{\sigma_e^2}{\kappa_j^{v^2} + \sigma_e^2}\right)(\mathbf{b}_j^{v^T}\mathbf{a}_i^v)^2 \quad (9)$$

A Method to Estimate Noise Variance

We must usually guess the values of noise variance when we use Wiener filter since we cannot know the values. Let consider how to estimate the noise variance below. As discussed above the MSE_{regr} for the noise free case can be computed if the spectral sensitivities of a set of color sensors, spectral reflectance of objects and spectral power distributions for recording and viewing illuminants are available and it is represented by

$$MSE_{regr} = \sum_{i=1}^{\alpha}\|\mathbf{a}_i^v\|^2 - \sum_{i=1}^{\alpha}\|P_{CV}\mathbf{a}_i^v\|^2. \quad (10)$$

If we let the noise variance $\sigma_e^2 = 0$ for Wiener filter in (2), the $MSE_{wien}(\sigma_e^2 = 0)$ for noisy signal by (1) is given

$$MSE_{wien}(\sigma_e^2 = 0) = \sum_{i=1}^{\alpha}\|\mathbf{a}_i^v\|^2 - \sum_{i=1}^{\alpha}\|P_{CV}\mathbf{a}_i^v\|^2 + \sum_{i=1}^{\alpha}\sum_{j=1}^{\beta}\left(\frac{\sigma_e^2}{\kappa_j^{v^2}}\right)(\mathbf{b}_j^{v^T}\mathbf{a}_i^v)^2, \quad (11)$$

where σ_e^2 represents the variance of noisy signals. Therefore, the noise variance σ_e^2 can be estimated from (10) and (11), and is formulated by

$$\sigma_e^2 = \frac{MSE_{wien}(\sigma_e^2 = 0) - MSE_{regr}}{\sum_{i=1}^{\alpha}\sum_{j=1}^{\beta}\frac{\left(\mathbf{b}_j^{v^T}\mathbf{a}_i^v\right)^2}{\kappa_j^{v^2}}} \quad (12)$$

In summarize, the noise variance can be estimated following: (1) Prepare a color chart, an image acquisition device and a recording illuminant of which spectral properties are known, (2) Compute the MSE_{regr} by using (10). (3) The $MSE_{wien}(\sigma_e^2 = 0)$ can be obtained by using (3) with the application of Wiener filter (with $\sigma_e^2 = 0$) to image signals averaging over color samples.

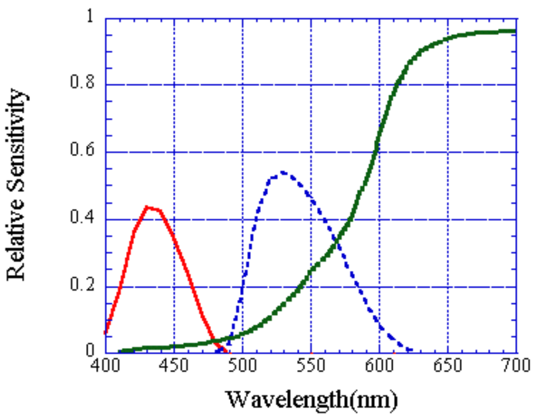
Experimental Results

To confirm the relationships between the MSE and the NIF, a Kodak Q60R1 color chart (228 colors), a Macbeth color checker (24 colors), a Munsell color chip set (641 chips), an offset printing sample set (216 colors), a set of patches by a thermal sublimation color printer (125 colors) and a set of natural objects including flowers, leaves, fruits, vegetables and human skins etc.(155 colors) were used to measure surface spectral reflectances and to calculate a signal from a set of sensors. Five sets of sensors were used to predict the noise influence on color correction. The spectral transmittance of the Kodak Wratten gelatin filters and the interference filters, and the spectral sensitivities of a CCD camera are represented in Fig.1. We call three filters which composed a set of interference filters with peak wavelengths of the 450, 550 and 610nm of the transmittance, and call the four filters which composed of the above set with a fourth filter of 500nm peak wavelength. Five filters are a set which includes all the filters shown in Fig.1(c). CIE D65, A and F2 were used as a recording and viewing illuminants.

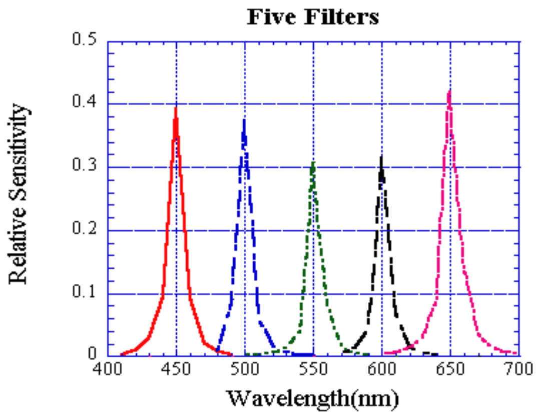
Simulated noisy responses from the sensors were computed using (1) for each reflectance using random numbers. Since the intensity of the signals is limited by practical limitations, it is necessary to include a constraint on the signal power. The constraint on the signal power can be expressed by [3]

$$\rho = Tr\left(S_L R_{SS} S_L^T\right) \tag{13}$$

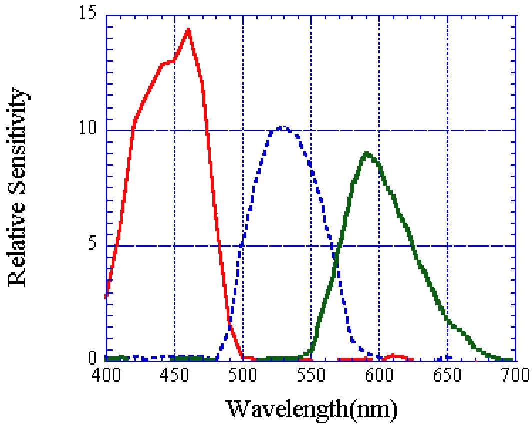
where ρ is a constant for each sensor set. The noise was assumed to be signal independent and zero-mean with variance σ_e^2. The SNR is defined by [3]



(a) Kodak Wratten Gelatin Filters



(b)Interference Filters



(c)CCD Sensors

Figure 1. Spectral sensitivities of sensor sets used in this paper

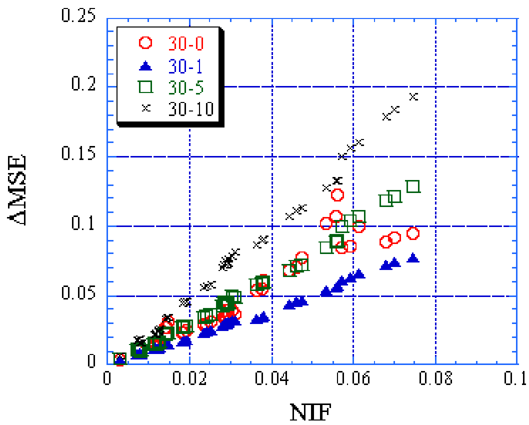


Figure 2. ΔMSE as a function of the NIF for five filters at 30dB. $\Delta\mathrm{MSE}_{\mathrm{wien}}\left(\sigma_e^2\right)$ *was computed with actual noise variance multiplied with zero, one, five and ten*

$$SNR = 10\log\left(\frac{Tr\left(S_L R_{SS} S_L^T\right)}{\sigma_e^2}\right). \tag{14}$$

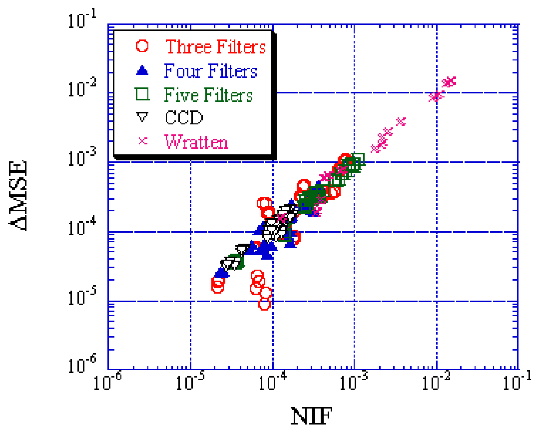
The noise variance (σ_e^2) for each SNR was computed by using (14). The magnitude of the random numbers was determined so that the variance of the random numbers coincides with that estimated by (14). The $\mathrm{MSE}_{wien}(\sigma_e^2)$ was computed by averaging the differences in fundamental vectors using (3) for all recording reflectances by the use of noisy signals. The increased MSE by the presence of noise, $\Delta\mathrm{MSE}$, is defined by $\Delta\mathrm{MSE} = \mathrm{MSE}_{wien}(\sigma_e^2) - \mathrm{MSE}_{regr}$. The values of the MSE_{regr} were computed by averaging for all colors using (3) for the noise free case, i.e. $\sigma_e^2 = 0$, and signals without noise. The values of NIF were computed using (9) for each combination of sensor sets, color charts and illuminants. Fig.2 shows a typical example of the relationship between the $\Delta\mathrm{MSE}$ and the proposed noise influence factor, NIF, using the five filters at 30dB. In Weiner estimation, we must guess a value of the noise variance since it is usually difficult to know the variance exactly. To compute the $\mathrm{MSE}_{wien}(\sigma_e^2)$ using (3) the noise variance σ_e^2 multiplied with zero, one, five and ten were used for Wiener estimation. This result indicates that the $\Delta\mathrm{MSE}$ increases linearly as the NIF increases for all noise variance multiplications and that the noise variance multiplied with one gives best results. Similar results were obtained for all sensor sets. From this result, it is important to estimate the noise variance correctly for the accurate color correction.

By using the procedures described above, noise variances were estimated. A typical example of the results is summarized in Table 1. The estimates were performed for CCD image sensors using Kodak Q60R1 under CIE F2 recording and D65 viewing illuminant. Actual noise variances were computed directly from the autocorrelation matrix of the random numbers. The results show that noise variances can be estimated accurately from the procedures described above.

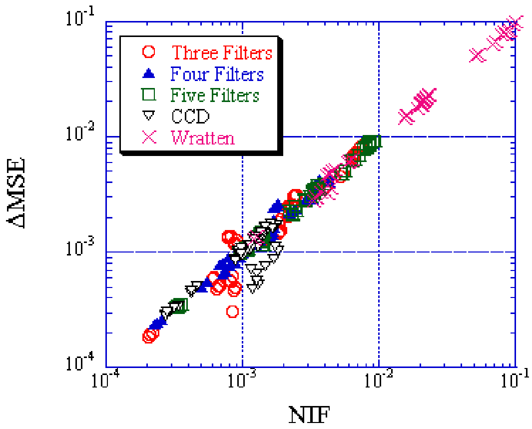
Figure 3 gives the relationships between the $\Delta\mathrm{MSE}$ and the NIF for each combination of sensor sets, color charts and illuminants at 50, 40 and 30dB. These results were obtained by using the noise variance that is estimated by the procedures described above. It is understood that $\Delta\mathrm{MSE}$ increases linearly as the NIF increases at all noise levels. The results indicate that the proposed noise influence factor is a good measure to predict the influence of noise on the accuracy of color correction. The $\Delta\mathrm{MSE}$ for a wratten gelatin filter set is always larger than that of other filters at all noise levels. This result is due to the small singular values of this filter set compared with others since small singular values make the values of the MSE large as seen from (7).

Table 1 Estimated Noise Variances at Various Noise Levels

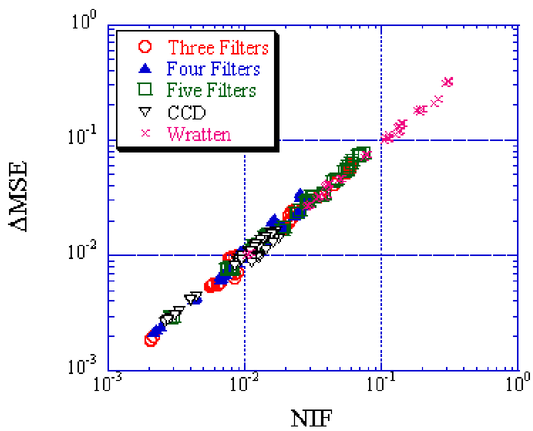
SNR	Actual Noise Variance	Estimated Noise Variance
30dB	9.981×10^{-4}	1.008×10^{-3}
40dB	9.980×10^{-5}	1.005×10^{-4}
50dB	9.801×10^{-6}	9.795×10^{-6}
60dB	9.800×10^{-7}	9.555×10^{-7}



(a)50dB



(b)40dB



(c)30dB

Figure 3. ΔMSE as a function of the NIF for each combination of sensor sets, color charts and illuminants

Conclusion

The influence of noise on the accuracy of color correction was studied based on a new simple formula for colorimetric evaluation of a set of color sensors. From computer simulations, it is confirmed that the proposed noise influence factor, the NIF, predicts the increased MSE by the presence of noise for various combinations of sensor sets, objects and illuminants at various noise levels. The NIF provides a good measure to predict the influence of noise on the accuracy of color correction.

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