The Fuzzy Integral as Similarity Measure for a New Color Morphology

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Abstract

Mathematical Morphology is a well-known discipline for the analysis of spatial structures which has found an extended application in image processing. The extension of grayscale morphology, which have been successfully used in multiple applications, to color morphology has very few examples in the literature.

While the maximum and the minimum, which are the basic concepts underlying a morphology, of a one-dimensional quantity is well defined, the definition of such operations for a multi-dimensional one is not trivial. In case of color values, expressed normally in three- or four-dimensional spaces, such a definition is complicated by the special features of human color perception. The here presented paper proposes a reduced morphology based on the fusion operator known as fuzzy integral. As a result of the developments a new type of so-called directed morphological operations is defined. Such operations allow the user to influence the result of the morphological operation by means of a color preference.

The obtained morphology has been applied on both synthetic and real application images with successful results.

1 Introduction

Mathematical Morphology is a well known discipline for the analysis of spatial structures which has found an extended application in image processing.[9,10] Under mathematical morphology for image processing a set of operations for different image-to-image transformations by means of a structuring element can be considered. This element, whose dimensions are normally smaller than the image being treated, acts as a structural probe for the suppression, enhancement or preservation of certain elements of the treated image.[10]

In spite of the uncertainty present in the definition of color, only one work[7] has till now considered the introduction of fuzzy concepts on the definition of such a morphology. The present paper introduces a new fuzzy color morphology based on the concept of aggregation operators as similarity measures presented in Ref. 2. The fuzzy integral is used as aggregation operator for solving the ranking operation on three-dimensional color vectors. Thus a reduced morphology is obtained. A very novel aspect of the here presented morphology is the possibility to define a new class of operations, which can be qualified as directed.

2 Mathematical Morphology

Two basic operations underlay mathematical morphology: dilation and erosion. Up to these basic operations a more or less complex universe of operators can be derived for the resolution of more or less complex tasks.

The mathematical definition of both operations for a grayvalue image I and a structuring element S is (⊕ states for a dilation, ⊗ for an erosion operation):

$$(I \oplus S)(z) = \max_{z=i+s}\{f(z) \mid i \in I, s \in S\} \tag{1}$$

$$(I \otimes S)(z) = \min_{z=i+s}\{f(z) \mid i \in I, s \in S\} \tag{2}$$

In the last time some works have been presented on the fuzzy generalization of grayscale morphology. These generalizations considered some degree of fuzziness for the treatment of uncertainty. For that purpose many fuzzy concepts have been used: T- and S-norms, the extension principle, fuzzy integral, fuzzy subsethood and OWAs.[8] Only one of these generalizations, presented in Ref. 7, considers the problem of allowing the morphological processing of color images.

At this point is worthwhile to remember the key ideas of Serra[9] for the generalization of mathematical morphology to other types of image data[8]:

- the existence of an order relationship,
- the existence of a supremum or infimum pertaining to that order,
- and the possibility of admitting an infinity of operands.

2 Color Morphology

Two basic problems arise when trying to conceive a mathematical morphology for color images. On the one hand the multidimensional representation of color. Most of image acquisition devices use a three- or even four-dimensional space for the representation of color. Thus the establishment of a maximum or minimum on this signal is much more complex than just applying a simple comparison between the pixels in the subset of a grayvalue image. Different strategies of multivariate ranking have been proposed for solving this problem.

On the other hand color definition is a subjective, complex and uncertain matter. Not only different features of color vision in human beings [1], including e.g. color constancy in front of changing illumination conditions or hue mutual contrast, supports this affirmation, but also multiple physiological facts. These range from the impairment to distinguish between green and red to little individual dependant differences in the sensitivity curves of the cones in the retina.[1] Thus it seems to be logical that a generalized color morphology considers some fuzzy concept.

Beyond the key ideas of Serra[9] for a generalized morphology, additional requirements were considered to be important for the establishment of morphological operations on color images in Ref. 7:

- No new color values should be introduced in the treated image.
- Dilation should be defined as an increasing operation, while erosion as a decreasing one.
- Through the consecutive appliance of two masks the same result should be obtained as that one obtained through the appliance of a sole mask resulting from their binary dilation.

$$(I \oplus S_1) \oplus S_2 = I \oplus (S_1 \oplus_B S_2) \qquad (3)$$

- A generalized color operation should become a grayscale one when being applied to grayvalue images.
- The operation should present convergence.

Thus not all multivariate ordering principles can lead to a color morphology. Three types of color morphologies were established in Ref. 7: marginal, reduced and Pareto-morphologies. Few color morphologies have been introduced till now.[7] Reference 3 presented a marginal morphology adequate for the processing of label coded images. A reduced morphology was explored in Ref. 4. Finally in Ref. 7, Fuzzy Pareto Morphology for color images was presented. In spite of the tight relationship between color and fuzziness, this one has been till now the only attempt to incorporate any fuzzy concept to a color morphology.

Taking as inspiration the work in Ref. 2 a new fuzzy color morphology based on reduced ranking is developed in the present paper. The concepts underlying this work are the usage of the fuzzy integral as reducing function, and the relationship between aggregation operators and similarity measures.

3 Aggregation as Similarity and the Fuzzy Integral

In Ref. 2 a common theoretical framework for aggregation operators in the context of fuzzy systems is presented. This work states that aggregation operators can be used to measure distances in metric spaces, where an intrinsic relationship exists between the aggregation operator and the distance function being used. Aggregation operators are used in fuzzy inference systems to establish the membership function of the output variable.[12]

A new interpretation to this operation is given in Ref. 2. Its result can be seen to measure the similarity to prototype elements of the output set, which receive the name of Ideals.[2] In the feature space the Ideal is represented by the point where all the features have a maximum value, which is 1 in the case of normalized features, and its counterpart, which receive the name of Anti-Ideal, by the origin (see Fig.1).

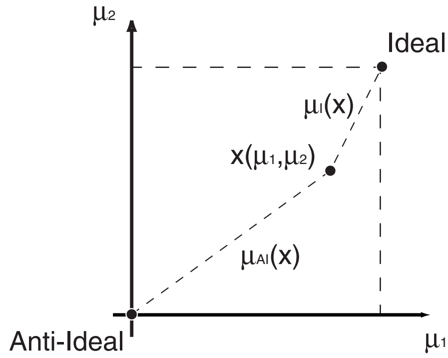

Figure 1. Representation of points Ideal and Anti-Ideal in the hypercube. μ_I: Membership function of the Ideal. μ_{AI}: Membership function of the Anti-Ideal.

The question arises if the Fuzzy Integral, as operator with positive properties for aggregation and unique one to allow the expression of interactions between integrands,[6] could be employed taking into consideration of the theoretical framework formerly elucidated.

The Theory of Fuzzy Measures, which constitutes the theoretical framework for the study of the Fuzzy Integral, was built on the conclusions made in Sugeno's pioneering work.[11] In an analogous way as fuzzy logic extended classical logic by adding subjectivity to the reason process, fuzzy measures consider the extension of classical measure theory with the introduction of subjectivity. In order to satisfy this attribute, classical measures (i.e. probability measures) are extended by relaxing their additivity axiom. I.e. fuzzy measures are functions on fuzzy sets satisfying the following conditions:

I. The fuzzy measure coefficients range from 0, fuzzy measure coefficient for the empty set, to 1, fuzzy measure coefficient for the set of all information sources.
II. Fuzzy measures are monotone increasing functions, what means that the fuzzy measure coefficient for a given set is always larger than the coefficient of any of its subsets.

Thus fuzzy measures include probability, possibility, necessity, belief and fuzzy-λ measures. Fuzzy measures are defined through its coefficients, which are used to evaluate the *a priori* importance of the different subsets of information sources ($\mu(A_i)$ in the expressions (3) and (4)). The establishment of a new kind of measures led to the establishment of a new integral, the fuzzy integral.

The fuzzy integral was introduced as a mathematical expression of the process of multiattribute fusion undertaken in the mind.[11] The basic idea of this approach is that when fusing information coming from different sources, the human being undertakes a subjective weighting of the different criteria or factors, and aggregates them in a non-linear fashion. Moreover the fuzzy integral considers the aggregation of fuzzified data and generalizes other aggregation operators as minimum, maximum, average, median, weighted sum, OWA, weighted minimum or weighted maximum.[6]

Two fuzzy integrals, known as Sugeno's and Choquet's Fuzzy Integral, are the most used in practical applications. The difference between them can be found in the used operators, what in the theory of fuzzy sets receives the name of T- and S-norms. While the Sugeno's Fuzzy Integral (S_μ) uses norms of the type maximum and minimum,

$$S_\mu[x_1,\ldots,x_n] = \bigvee_{i=1}^{n}[x_{(i)} \wedge \mu(A_{(i)})] \quad (3)$$

the Choquet's Fuzzy Integral (C_μ) was introduced as an extension in the framework of Fuzzy Measures Theory of the classical Lesbesgues's Integral.

$$C_\mu[x_1,\ldots,x_n] = \sum_{i=1}^{n}\left\{[x_{(i)} - x_{(i-1)}]\mu(A_{(i)})\right\} \quad (4)$$

For a deeper description of the expressions and the algorithmical details the reader is referred to Ref. 6.

4 New Color Morphology

When operating with a grayscale morphology the maximum and minimum of the ranking scheme are well defined, i.e. the pixel values of the images range from 0 to g_{max}, are natural numbers and one-dimensional. Normally the maximum value is the color white and the minimum the black one. In the case of multi-dimensional magnitudes, as color, such definition can not be trivially established.

Reduced color morphologies, as the one here presented, used a function of the different scalars in the color vector in order to realize the ranking scheme necessary for the definition of morphological operations. The usage of the fuzzy integral as an aggregation operator in the theoretical framework described in the former section allows to identify the maximum and the minimum with the Ideal and the Anti-Ideal. Thus the fuzzy integral can be used to measure the similarity to these points. Since the fuzzy integral generalizes most of known aggregation operators, the utilization of this operator improves the flexibility. The generalization of the fuzzy integral is achieved by the parameterization of the operator through the fuzzy measure coefficients. The usage of the fuzzy measures allows the implementation of so-called directed morphological operations as well.

The existence of directed operations is a novelty in morphological operations. By directed is meant that pixels with certain proximity to the Ideal (or the Anti-Ideal) would be favored (or not) in the ranking result through the values of the fuzzy measure coefficients. In this way one could speak of a red-dilation, a dilation where the red tones would be favored, or a blue-erosion. In these directed operations a color with certain preference is defined through the coefficients of the fuzzy measure. E.g a red-dilation would be accomplished if the coefficients of the subsets where the red channel appears are larger than the other ones (see Fig. 2).

The fuzzy measure described is used for the computation of the distance to the Ideal, while the so-called dual fuzzy measure is used for the Anti-Ideal. A detailed description of this concept would complicate excessively the scope of this communication. However the dual fuzzy measure can be intuitively understood as the complementary measure and is computed through the expression:

$$\mu^*(A) = 1 - \mu(A^c) \quad (5)$$

where (A^c) is the complement set of A. More information can be found in Ref. 6.

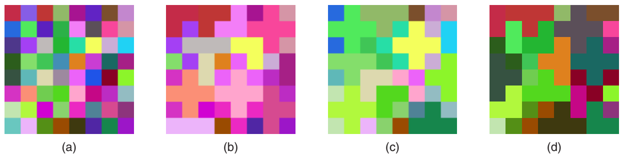

Figure 2. Exemplary usage of the here presented directed morphological operations on a synthetic image (a), where the color values of the pixels have been randomly defined. The results are obtained with a cross-shaped structuring element. (b) Result of red-dilation with following fuzzy measure: μ{R}=128, μ{G}=1, μ{B}=1, μ{RG}=200, μ{RB}=200, μ{GB}=1, μ{RGB}=255. (c) Green-dilation with μ{R}=1, μ{G}=128, μ{B}=1, μ{RG}=200, μ{RB}=1, μ{GB}=200, μ{RGB}=255. (d) Blue-erosion with μ{R}=1, μ{G}=1, μ{B}=128, μ{RG}=1, μ{RB}=200, μ{GB}=200, μ{RGB}=255.

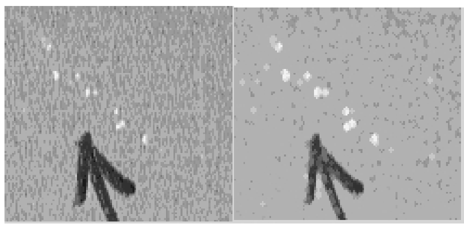

Figure 3. Pre-processing of a textile image with a yellow-directed dilation based on a cross-shaped mask.

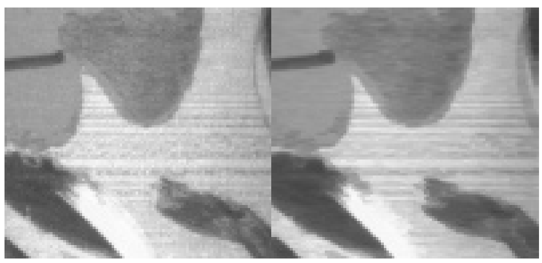

Figure 4. Pre-processing of a textile image with a brown-directed erosion based on a horizontal line shaped mask.

The procedure for a morphological operation can be summarized as following. The pixels underlying the structuring elements are selected. The distances to the Ideal and the Anti-Ideal are computed. The color of the pixel under the central element of the mask is substituted with that color of the pixel presenting a minimal distance to the Ideal and a maximal distance to the Anti-Ideal in case of a dilation. In an erosion operation the minimal distance to the Anti-Ideal and the maximal distance to the Ideal is considered.

4 Results

The defined morphology was employed in the pre-processing stage of a system for the automated visual inspection of textiles. The results can be observed in Figs.3-4. In this case the application of morphological operations directed to the color of the faults succeed in enhancing its visualization.

On the depicted images the inspection system was expected to detect a defect produced in the printing production stage. In then first case (Fig.3) the image was dilated through a cross shaped mask. On the other image (Fig. 4) a brown-directed erosion was used based on a horizontal line shaped mask. The values of the color whereby the fault is visualized were used as the fuzzy measure coefficients in both cases.

Conclusions

A color morphology was presented. This new morphology is based on the utilization of the fuzzy integral for the computation of similarities in the multi-dimensional space of color values. The computed similarities are used for the establishment of the ranking scheme needed in the definition of morphological operations.

The parameterization of the fuzzy integral through the fuzzy measure coefficients allows coping with the uncertainty produced by the vectorial nature of color information. Furthermore the fuzzy measures are used to express subjective *a priori* importance of the different color channels. This fact enables the expression of color preferences by the user of the color morphology in form of so-called directed operations. When applying the ranking scheme the color to which the operation is directed will be considered to be closer to the prototype points Ideal or Anti-Ideal than color values with similar vector norm.

The new defined directed operations were successfully used on both synthetic and real application images. Our work in the future will consider the characterization of the here presented morphology concerning the type of used fuzzy integral and the realization of more complex operations.

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Biography

Aureli Soria-Frisch was born in Barcelona, Catalonia in 1969. He studied electrical engineering at the Politechnical University of Catalonia and in 1995 he wrote his final thesis at the Humboldt University Berlin on the area of speech recognition. At the present time he is writing his Doctoral Degree Thesis at the Fraunhofer IPK, which was sponsored by a grant of La Caixa Foundation and the German exchange organization D.A.A.D. The thesis treats fuzzy fusion operators in Pattern Recognition. His research interests are biological-based algorithms for computer vision, the fuzzy logic and fuzzy measure theories, and multisensory data fusion.

Mario Köppen studied physics at the Humboldt University of Berlin and received his Diploma in solid state physics in 1991. Afterwards, he worked as research assistant at the Central Institute for Cybernetics and Information Processing in Berlin and changed his main research interests on image processing and neural networks. Since 1992, he is with the Fraunhofer Institute for Production Systems and Design Technology. He continued his works on image processing, in particular on the application of Soft Computing in image processing. Mario Köppen has finished his Doctoral Degree Thesis with the title "Development of a learning computer vision system through Soft Computing".