The Kubelka-Munk Theory for Color Image Invariant Properties

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Abstract

A fundamental problem in color image processing is the integration of the physical laws of light reflection into image processing results, the problem known as photometric invariance. The derivation of object properties from color images yields the extraction of geometric and photometric invariants from color images. Photometric invariance is to be derived from the physics of reflection. In this paper, we rehearse the results from radiative transfer theory to model the reflection and transmission of light in colored layers. We concentrate on the Kubelka-Munk theory for colored layers, which is posed as a general model for color image formation. The model is used for decades in the painting and printing industrie, and is proven to be valid for a wide range of materials. We relate the Kubelka-Munk theory to photometric models currently used in image processing. As a consequence, the wide range of materials for which Kubelka-Munk is proven valid may be inherited to algorithms based on newer models. Furthermore, photometric invariant properties proven for one model are, by using Kubelka-Munk, easily extended to related models.

Introduction

Color seems to be an unalienable property of objects. It is the orange that has that color. However, the heart of the matter is quite different. Human perception actively assigns colors to an observed scene. There is a discrepancy between the physics of light, and color as signified by the brain. It is evolution that has shaped the actual mechanism of color vision. Evolution, such that a species adapts to its (physical) environment, has driven the use of color by perception.

In terms of physics, daylight is reflected by an object and reaches the eye. It is the reflectance ratio over the wavelengths of radiant energy that is an object property, hence the reflection function for an orange indeed is a physical characteristic of the fruit. However, the amount of radiant energy falling onto the retina depends on both the object reflectance function, the geometry of the object, and the light source illuminating the object. Still, we observe an orange to be orange in sunlight, by candlelight, independent of shadow, frontal illumination, or oblique illumination.

lumination. All these variables influence the energy distribution as it enters the eye, the variability being imposed by the physical laws of light reflection. Human color vision has adapted to include these physical laws, due to which we neglect the scene induced variations.

From a computer vision perspective, a fundamental question is how to integrate the physical laws of light reflection into color measurement? Modeling the physical process of color image formation provides a clue to the object-specific parameters [3, 15, 17, 19, 20, 22, 25, 28, 31, 32]. The question boils down to deriving the invariant properties of color vision, [1, 4, 7, 8, 11, 14, 28]. With *invariance* we mean a property f of object t which receives value f(t) regardless unwanted conditions W in the appearance of t. For human color vision, the group of disturbing conditions W' are categorized by *shadow*, highlights, light source, and scene geometry. Scene geometry is determined by the number of light sources, light source directions, viewing direction, and object shape. The invariant class W' is referred to as photometric invariance. For observation of images, geometric invariance is of importance [6, 13, 16, 21, 30]. The group of spatial disturbing conditions is given by translation, rotation, and observation scale. Since the human eye projects the three-dimensional world onto a two-dimensional image, the group may be extended with projection invariance. Both photometric and geometric invariance are required for a color vision system to reduce the complexity intrinsic to color images [10].

Photometric invariance is to be derived from the physics of reflection. Recently, several papers have addressed the extraction of photometric models for computer vision or visual inspection tasks [9, 14, 22, 23, 28], probably the most famous model given by Shafer [28]. In astrophysics, the physics of light interacting with material is well established, the field known as radiative transfer [2, 29], initiated by the pionering work of Schuster [27]. In the field of colorimetry, the theories were rediscovered by Kubelka and Munk [20] when they described the reflection and transmission of light in colored layers. The theory of Kubelka-Munk has become famous in textile and dye industry. Note that the original problem solved by Schuster [27] is still of interest for computer vision [23, 24].

The Kubelka-Munk theory models the reflected and

transmitted spectrum of a colored layer, based on a material dependent scattering and absorption function. The theory unites spectral color formation for both reflecting materials as well as transparent materials into one photometric model. The theory has proven to be successful for a wide variety of materials and applications [15, 31]. Therefore, the Kubelka-Munk theory is well suited for determining material properties from color measurements. The theory combines Lambertian reflectance [31], Shafer's dichromatic reflectance [28], and Lambert-Beer transmissive absorption.

In this paper, we derive these well known reflection models from the Kubelka-Munk theory in order to generalize photometric invariant properties as given in e.g. [11, 10, 5]. The organization of the paper is as follows. In Section 2, the Kubelka-Munk theory is derived. Image formation for the case of reflecting surfaces is given in Section 2.1. For the case of light transmission through a colored layer, image formation is modelled in Section 2.2. Well known special instances of the models are discussed in Section 2.3. The theory is extended to true diffusion in Section 2.4. We conclude with an discussion of which invariant properties hold valid under these special instances.

Kubelka-Munk Theory and Image Formation

Transfer of light through a medium is characterized by three fundamental processes: absorption, scattering, and emission. Absorption is the process by which radiant energy is transformed into another form of energy, e.g. heat or light of different wavelength (fluorescence). Hence, radiant energy is lost. Scattering is the process by which the radiant energy is diffused into different directions. Emission is the process by which new radiant energy is created, e.g. by a light source inside the medium, or due to fluorescent properties of the medium. Transfer theory [2, 29] deals with the combined effects of these processes in a medium with spatial extent. The Kubelka-Munk theory models the effect of these processes under the assumption of a one-dimensional light flux, hence isotropic scattering within the material. [15, 20, 19, 31]. Under this assumption, the material layer is characterized by a wavelength dependent scatter coefficient and absorption coefficient. The class of materials for which the theory is useful ranges from dyed paper and textiles, opaque plastics, paint films, up to enamel and dental silicate cements [15]. The model may be applied to both reflecting and transparent material.

Color Formation for Reflection of Light

Consider a homogeneously colored material patch of uniform thickness d and infinitesimal area, characterized by its absorption coefficient $k(\lambda)$ and scatter coefficient $s(\lambda)$. When illuminated by incident light with spectral distribution $e(\lambda)$, light scattering within the material causes dif-



Figure 1: Illustration of the photometric model. The object, refractive index n_2 , is illuminated by $e(\lambda)$ (medium refractive index n_1), and light is reflected and scattered in the viewing direction.

fuse body reflection (Figure 1), while Fresnel interface reflectance occurs at the surface boundaries.

When the thickness of the layer is such that further increase in thickness does not affect the reflected color, Fresnel reflectance at the back surface may be neglected. The incident light is partly reflected at the front surface, and partly enters the material, is isotropically scattered, and a part again passes the front-surface boundary. The reflected spectrum in the viewing direction \vec{v} , ignoring secondary scattering after internal boundary reflection, is given by [15, 31]:

$$E_R(\lambda) = e(\lambda) \left(1 - \rho_f(\lambda, \vec{n}, \vec{s}, \vec{v})\right)^2 R_\infty(\lambda) + e(\lambda)\rho_f(\lambda, \vec{n}, \vec{s},)\vec{v} \quad (1)$$

where \vec{n} is the surface patch normal and \vec{s} the direction of the illumination source, and ρ_f the Fresnel front surface reflectance coefficient in the viewing direction. The body reflectance

$$R_{\infty}(\lambda) = a(\lambda) - b(\lambda) \tag{2}$$

depends on the absorption and scattering coefficient by

$$a(\lambda) = 1 + \frac{k(\lambda)}{s(\lambda)}, \quad b(\lambda) = \sqrt{a(\lambda)^2 - 1}$$
 (3)

Simplification is obtained by considering neutral interface reflection, assuming that the Fresnel reflectance coefficient has a constant value over the spectrum. For commonly used materials, interface reflection is constant with respect to wavelength within a few percent across the visible spectrum [15, 26]. Equation (1) reduces to

$$E_R(\lambda) = e(\lambda) \left(1 - \rho_f(\vec{n}, \vec{s}, \vec{v})\right)^2 R_\infty(\lambda) + e(\lambda)\rho_f(\vec{n}, \vec{s}, \vec{v}).$$
(4)

The influence of the Fresnel reflectance varies from perfectly diffuse body reflectance $\rho_f = 0$, or Lambertian reflection, to total mirroring of the illuminating source $(\rho_f = 1)$. Hence, the spectral color of E_R is an additive mixture of the color of the light source and the perfectly diffuse body reflectance color.

Because of projection of the energy distribution on the image plane, vectors \vec{n} , \vec{s} and \vec{v} will depend on the position at the imaging plane. The energy of the incoming spectrum at a point \vec{x} on the image plane is then related to

$$E_R(\lambda, \vec{x}) = e(\lambda, \vec{x}) \left(1 - \rho_f(\vec{x})\right)^2 R_\infty(\lambda, \vec{x}) + e(\lambda, \vec{x})\rho_f(\vec{x})$$
(5)

where the spectral distribution at each point x is generated off a specific material patch.

The major assumption made for the model of Eq. (5)is that locally planar surface patches are examined, for which the material is homogeneously colored. These constraints are imposed by the Kubelka-Munk theory, resulting in isotropic scattering of light within the material. The assumption is valid when the resolution is fine enough to consider locally uniform colored patches, whereas individual staining particles are not resolved. Further, the thickness of the layer is assumed to be such that no light reaches the other side of the material. For every day scenes, these assumptions seems to be justified. Concerning the Fresnel reflectance, the photometric model assumes a neutral interface at the surface patch. As discussed in [26, 28], deviations of ρ_f over the visible spectrum are small for commonly used materials, therefore the Fresnel reflectance coefficient may be considered constant. The internally Fresnel reflected light contributes little in many cases [31], and is ignored in the model.

Color Formation for Transmission of Light

Consider a homogeneously colored material patch of uniform thickness d and infinitesimal area, characterized by its absorption coefficient $k(\lambda)$ and scatter coefficient $s(\lambda)$. When illuminated by incident light with spectral distribution $e(\lambda)$, absorption and scattering by the material determines its transmission color (Figure 2), while Fresnel interface reflectance occurs at both the front and back surface boundaries.

When the layer is thin, such that the material is transparent, the transmitted spectrum through the layer in the viewing direction \vec{v} , ignoring the effect of interreflections between the material surfaces, is given by [15, 31]:

$$E_T(\lambda) = \frac{e(\lambda) \left(1 - \rho_f(\lambda, \vec{n}, \vec{s}, \vec{v})\right) \left(1 - \rho_b(\lambda, \vec{n}, \vec{s}, \vec{v})\right) b(\lambda)}{D}$$
(6)

where

$$D = a(\lambda) \sinh[b(\lambda)s(\lambda)l(\vec{n},\vec{s},\vec{v})c] + b(\lambda) \cosh[b(\lambda)s(\lambda)l(\vec{n},\vec{s},\vec{v})c] .$$

Again, \vec{n} is the material patch normal and \vec{s} is the direction of the illumination source. Further, c is the staining concentration and l the distance traveled by the light through



Figure 2: Illustration of the photometric model. The object, refractive index n_2 , is illuminated by $e(\lambda)$ (medium refractive index n_1). When the material is transparent, light is transmitted through the material, enters medium n_3 , and is observed.

the material. The terms ρ_f and ρ_b denote the Fresnel front and back surface reflectance coefficient, respectively. The factors *a* and *b* depend on the absorption and scattering coefficients as given by Eq. (3).

Simplification is obtained by considering neutral interface reflection, assuming that the Fresnel reflectance coefficients have a constant value over the spectrum. In that case, the Fresnel reflectance affects the intensity of the transmitted light only. Further, by considering a small angle of incidence at the transparent layer, the path length $l(\vec{n}, \vec{s}, \vec{v}) = d$. Equation (6) reduces to

$$E_T(\lambda) = \frac{e(\lambda) \left(1 - \rho_f(\vec{n}, \vec{s}, \vec{v})\right) \left(1 - \rho_b(\vec{n}, \vec{s}, \vec{v})\right) b(\lambda)}{a(\lambda) \sinh b(\lambda) s(\lambda) dc] + b(\lambda) \cosh[b(\lambda) s(\lambda) dc]} .$$
(7)

Because of projection of the energy distribution on the image plane, vectors \vec{n} , \vec{s} and \vec{v} will depend on the position \vec{x} at the imaging plane,

$$E_T(\lambda, \vec{x}) = \frac{e(\lambda, \vec{x})(1 - \rho_f(\vec{x}))(1 - \rho_b(\vec{x}))b(\lambda, \vec{x})}{D'}$$
(8)

where

$$D' = a(\lambda, \vec{x}) \sinh[b(\lambda, \vec{x})s(\lambda, \vec{x})d(\vec{x})c(\vec{x})] + b(\lambda, \vec{x}) \cosh \phi(\lambda, \vec{x})s(\lambda, \vec{x})d(\vec{x})c(\vec{x})] .$$

The spectral distribution at each point x is generated off a specific transparent patch.

One of the assumptions made for the model of Eq. (8) is that locally planar material patches are examined, with parallel sides, for which the material is homogeneously colored. The assumption is valid when the material is non-fluorescent nor in any sense optically active, and the resolution is fine enough to consider locally uniform colored patches, while individual stain particles are not resolved.

Again, these constraints are imposed by the Kubelka-Munk theory. Further, normal incidence of light at the layer is assumed, so that the optical path length through the layer approximates its thickness. In transmission light microscopy, the preparation and observation conditions fairly justify these assumptions. Concerning the Fresnel reflectance, the photometric model assumes a neutral interface at the transparent patch. As discussed in [26], deviations of ρ_f , ρ_b over the visible spectrum are small for commonly used materials. For example, the refractive index of immersion oil often used in microscopy only varies 3.3% over the visible spectrum. Therefore, the Fresnel reflectance coefficients ρ_f and ρ_b may be considered constant over the spectrum. The contribution of internally Fresnel reflected light is small in many cases [31], and is therefore ignored in the model.

Special Cases

Thus far, we have achieved a photometric model for spectral color formation, which is applicable for both reflecting and transmitting materials, and valid under a wide variety of circumstances and materials. The following special cases can be derived.

For matte, dull surfaces, the Fresnel coefficient can be considered neglectable, $\rho_f(\vec{x}) \approx 0$, for which E_R Eq. (5) reduces to the Lambertian model for diffuse body reflection,

$$E_R(\lambda, \, ec{x}) = e(\lambda, \, ec{x}) R_\infty(\lambda, ec{x})$$

(9)

as expected.

By introducing $c_b(\lambda) = e(\lambda)R_{\infty}(\lambda)$, $c_i(\lambda) = e(\lambda)$, $m_b(\vec{n}, \vec{s}, \vec{v}) = (1 - \rho_f(\vec{n}, \vec{s}, \vec{v}))^2$, and $m_i(\vec{n}, \vec{s}, \vec{v}) = \rho_f(\vec{n}, \vec{s}, \vec{v})$, Eq. (4) may be reformulated as

$$E_R(\lambda) = m_b(\vec{n}, \vec{s}, \vec{v})c_b(\lambda) + m_i(\vec{n}, \vec{s}, \vec{v}c_i(\lambda)$$
(10)

which corresponds to the dichromatic reflection model proposed by Shafer [28].

For light transmission, when the scattering coefficient is low compared to the absorption coefficient, $s(\lambda) \ll k(\lambda)$, E_T Eq. (8) reduces to Bouguer's or Lambert-Beer's law for absorption [31],

$$E_T(\lambda, \vec{x}) = e(\lambda, \vec{x}) \left(1 - \rho_f(\vec{x})\right) \left(1 - \rho_b(\vec{x})\right)$$
$$exp\left(-k(\lambda, \vec{x})d(\vec{x})c(\vec{x})\right) \quad (11)$$

as expected.

Further, a unified model for both reflection and transmission of light is obtained when considering Lambertian reflection and a uniform illumination for both cases. For matte, dull surfaces, and a uniform illumination affected by shading, E_R Eq. (5) reduces to a multiplicative (Lambertian) model for body reflection,

$$E_R(\lambda, \vec{x}) = e(\lambda)i(\vec{x})R_\infty(\lambda, \vec{x})$$
(12)

where $e(\lambda)$ is the colored but spatially uniform illumination and $i(\vec{x})$ denotes the intensity distribution due to the surface geometry. Similar, for a uniform illuminated transparent material, intensity affected by shading and Fresnel reflectance, E_T Eq. (8) may be rewritten as

$$E_T(\lambda, \vec{x}) = e(\lambda)i(\vec{x})C(\lambda, \vec{x})$$
(13)

where $i(\vec{x})$ denotes the intensity distribution, including Fresnel reflectance at front and back surface. Further, $C(\lambda, \vec{x})$ represents the total attenuation at \vec{x} .

Locally Non-uniform Colored Material

For the case of a spatially varying absorption and reflectance function, such that the variation scale is small compared to the optical thickness of the material, we have to consider a true three-dimensional scattering and absorption flux inside the material. The scattered light energy at a given point inside the material causes a diffuse flux throughout the material. The diffusion process is given by the Milne-Eddington law [12],

$$\frac{1}{3}\nabla^2 J = \kappa(\lambda)^2 \left(\lambda J - 4\pi S\right) \tag{14}$$

where $\kappa(\lambda) = k(\lambda) + s(\lambda)$ is the total extinction coefficient, and S is the source function:

$$4\pi S = \kappa(\lambda) I(\lambda) e^{-\kappa(\lambda)s}$$
(15)

where s is the path length to the point inside the medium. The theory is related to computer vision by Koenderink and van Doorn [18], illustrated for the case of shading by translucent objects.

Conclusions

The derivation of object properties from color images yiels the extraction of geometric and photometric invariants from color images. Modeling the physical process of color image formation gives insight into the disturbing conditions during image acquisition. Hence, provides the calculation of photometric invariance.

Combining the results of section Section 2.3 with photometric invariants as derived in [11, 10, 5] leads to the following conclusions. Hue is only useful for reflectance of light, where specularities may occur. For transmission of light (Lambert-Beer), too much information is compressed. Chromaticity (r/(r + g + b), g/(r + g + b))is useful in both light reflectance and transmittance. For reflectance, chromaticity is invariant to shadow and shading, whereas for transmittance, chromaticity is invariant to stain intensity and dye thickness. Color edge strength is invariant to the illumination color for both Lambertian reflection and Lambert-Beer absorption, not for dichromatic reflection. Invariant results valid under one model can be transferred to -by Kubelka-Munk- related models, allowing for a well founded choice between various photometric invariants at hand.

For colored layers, the Kubelka-Munk theory describes the light reflected by the material. The model unites both reflectance of light and transparent materials. The class of materials for which the theory is useful ranges from dyed paper and textiles, opaque plastics, paint films, up to enamel and dental silicate cements [15]. The theory combines Lambertian reflectance [31], Shafer's dichromatic reflectance [28], and Lambert-Beer transmissive absorption.

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Biography

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