COLOR IMAGE RESTORATION

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ABSTRACT

In this paper, we present a new method for color images restoration using PDE. A general framework of anisotropic diffusion of color images is presented. Severel results applied on synthetic and real images shows that our model is able to conrol the diffusion in the neigboring of the edges and corners.

Key word : Color image restoration, PDE, Anisotropic diffusion.

1. INTRODUCTION

Since a few years, several methods based on partial differential equation have been developed for grey level image restoration. The start of the design of PDE schemes in image restoration has been possible by the Scale Space formalization [6]. Then an axiomatization of multi-scale analysis has been proposed by Alvarez and al. [1].

The anisotropic diffusion idea is to smooth the image in the direction parallel to object boundaries and prevent, as much as posssible, diffusion across edges. Among the models of anisotropic diffusion for grey level image restoration, we find Malik and Perona[7]; L. Alvarez, P.L. Lions and J.M. Morel[2]. For color image restoration, several methods exist but there is no well defined theory giving a basis for the quantification of these approaches, and often results have to be visualy quantified.

We show in this paper that existing restoration models suffer from several problems arisen by the way in which the vectorial information is taken into account. There is mainly two approaches to handle color data.
The first and simpler approach consists in dealing with each color component separately and identically, this way does not generally produce satisfactory result.
The second one consists in taking into account pixel vectorial information in a unique restoration scheme.

We are interested in this paper in the vectorial aspect of color in RGB space and not in the color by itself. We propose a new approch for color image restoration based on PDE. We show here that our approch control very well the diffusion near edge and corner points.

In this paper, we first present in section 2 a general framework based on PDE for scalar image restoration. In section 3 we will present geometry of color RGB space in order to estimate local variations such as gradient and curvature. In section 4, we will present and discuss common aproches for color image retoration. Then in section 5, we introduce our method based on PDE for color image restoration. This method is compared with others approaches. Various results applied on synthetic and real images are presented.

2. PDE AND ANISOTROPIC DIFFUSION

in the following the image will be represented as a function defined as : $I \ : \ \Re^2 \ - \to \Re^d$.
The case where $d = 1$ corresponds to grey level images, the case $d = 3$, corresponds to color images. In this section we will consider the case $d = 1$.

2.1. Principle of Anisotropic Diffusion

Scalar image restoration consists in minimising the following functional :

$$E(I) \ = \ \frac{1}{2} \parallel I - P\, I_0 \parallel \ + \lambda \int_\Omega \Phi\left(\|\nabla I\|\right) d\Omega \qquad (1)$$

The minimization can be writen via the following equation coming from the Euler-Lagrange equations :

$$P^* \left(I - P\, I_0\right) \ + \ \lambda \ \text{div} \left(\Phi^{'} \left(\|\nabla I\|\right) \frac{\nabla I}{\|\nabla I\|}\right) \ = \ 0 \qquad (2)$$

which can be also writen as :

$$P^* \left(I - P\, I_0\right) \ + \ \lambda \left(\Phi^{''} \left(\|\nabla I\|\right) I_{\xi\xi} \ + \ \Phi^{'} \left(\|\nabla I\|\right) \frac{\nabla I}{\|\nabla I\|} I_{\eta\eta} \right) \ = \ 0 \qquad (3)$$

with :

- P^* is the “adjoint” operator of P.
- div : the divergence operator.

The parameter λ must to be greater than 0 .
The anisotropic PDE evolution coming from this minmization can be writen as :

$$\frac{\partial I}{\partial t} = c_\xi \ I_{\xi\xi} + c_\eta \ I_{\eta\eta} \tag{4}$$

with :

$$\begin{array}{c} c_\xi = \Phi^{''}(\|\nabla I\|) \ and \ c_\eta = \Phi^{'}(\|\nabla I\|) \frac{\nabla I}{\|\nabla I\|} \\ \eta = \frac{\nabla I}{\|\nabla I\|} \ et \ \xi \perp \eta \end{array} \tag{5}$$

The diffusion becomes anisotropic when a direction of diffusion is favored regardless to another direction. Here it is clear that the diffusion depends on the function Φ, for example, if $\Phi(x) = x^2$, the equation 4 corresponds to the classical heat equation.
Many models based on PDE for scalar image restoration can be written as the equation 4. Amoung these models, we find the Scale-Space of Malik and Perona [7], a selectif smoothing model proposed in [2] and Shock filter used for debluring scalar image [8].

3. COLOR IMAGE SEGMENTATION

In this section, we will now consider $d = 3$.

3.1. Vectorial Geometry

We denote I^i the i^{th} component for I $(i \epsilon \{R, G, B\})$ i.e:

$$I(x,y) = \begin{pmatrix} R(x,y) = I^1(x,y) \\ G(x,y) = I^2(x,y) \\ B(x,y) = I^3(x,y) \end{pmatrix}$$

The riemannian geometry framework for edge detection in vector-valued images has been first proposed by DiZenzo[4]. Let us consider the total differential of the image

$$dI = \frac{\partial I}{\partial x} dx + \frac{\partial I}{\partial y} dy \tag{6}$$

Then we are interested in the norm of this vector :

$$\|dI\|^2 = \begin{bmatrix} dx \\ dy \end{bmatrix}^T \begin{bmatrix} g_{11} & g_{12} \\ g_{12} & g_{22} \end{bmatrix} \begin{bmatrix} dx \\ dy \end{bmatrix} \tag{7}$$

Where g_{ij} is known as the multi-spectral tensor :

$$g_{ij} = \frac{\partial I}{\partial xi} \cdot \frac{\partial I}{\partial xj}$$

The two eigenvalues of g_{ij} are the extremum of $\|dI\|^2$, and the orthogonal egenvectors η, ξ are the corresponding variation directions:

$$\begin{cases} \lambda \pm = \frac{g_{11} + g_{22} \pm \sqrt{(g_{11} - g_{22})^2 + 4 g_{12}{}^2}}{2} \\ \eta = \arctan \frac{\lambda_+ - g_{11}}{g_{12}} \\ \xi = \eta + \frac{\pi}{2} \end{cases}$$

In our approach of color image restoration, we will also make use of a “color curvature” measure [5], in order to control the diffusion at corner points. The color curvature $\mathcal{K}$ can be defined as :

$$\mathcal{K} = \frac{d\theta}{ds} \tag{8}$$

with s is the curvulinear X-coordinate. The equation 8 can be written as :

$$\kappa = \frac{\partial \theta}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial \theta}{\partial y} \cdot \frac{\partial y}{\partial s} \tag{9}$$

If we developp a first part of the expression 9, we obtaind the following expression for $\frac{\partial \theta}{\partial x}$:

$$\frac{\partial \theta}{\partial x} = \frac{\begin{array}{l} \Big(A B_x B_{xy} + B B_{xx} B_y - 2 V_x V_{xy} V_y{}^2 + 2 B_x{}^2 B_{xx} B_y + \\ 2 B_x B_{xy} B_y{}^2 - 2 B_x B_y R_x R_{xx} + A R_x R_{xy} - 2 B_x B_{xx} R_x R_y + \\ 2 B_{xy} B_y R_x R_y + A R_{xx} R_y - 2 R_x{}^2 R_{xx} R_y + 2 B_x B_y R_{xy} R_y + \\ 2 R_x R_{xy} R_y{}^2 - 2 B_x B_y V_x X_{xx} - 2 R_x R_y V_x X_{xx} + A V_x V_{xy} - \\ 2 B_x B_{xx} V_x V_y + 2 R_x R_{xx} V_x V_y + 2 B_{xy} B_y V_x V_y - A X_{xx} V_y - \\ 2 V_x{}^2 X_{xx} V_y + 2 R_{xy} R_y V_x V_y + 2 B_x B_y V_{xy} V_y + 2 R_x R_y V_{xy} V_y \Big) \end{array}}{A^2 + 4 \Big(B_x B_y + R_x R_y + V_x V_y \Big)^2} \tag{10}$$

with : $A = B_x{}^2 - B_y{}^2 + R_x{}^2 - R_y{}^2 + V_x{}^2 - V_y{}^2$.

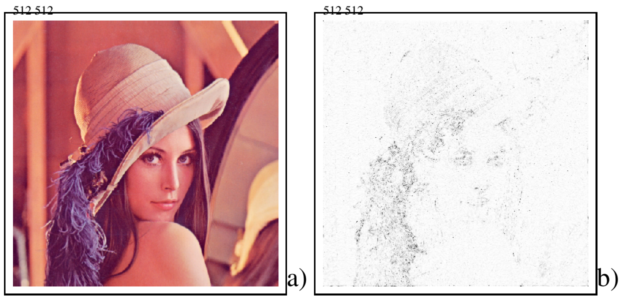

Fig. 1. *a) real color image (512×512), b) Color curvature (gaussian filter, $\sigma = 1.0$)*

3.2. Color gradient norms

Considering the expressions that have been exposed at the previous section, several standart norms can be defined. A direct extension of the definition of the vectorial analysis is to choose the gradient norm : $N = \sqrt{\Lambda_+}$. Note that for $d = 1$, $\Lambda_+ = \|\nabla I\|^2$, $\Lambda_- = 0$ and $(\cos\theta_+ , \sin\theta_+) = \frac{\nabla I}{\|\nabla I\|}$.

Sapiro proposes in [9] to choose $N = f(\Lambda_+ - \Lambda_-)$, however, he did not recommended a specific f which must be a nondecresing function of Λ_+ and Λ_-. In his opinion, edges in the vectorial case are not caracterised by a large lambda but by $\Lambda_+ >> \Lambda_-$. He proposes to use the function $f(.) = \sqrt{\Lambda_+ - \Lambda_-}$.

Blomgren propose in [3] a new definition of a gradient norm : the total variation norm for vector-valued function that can be applied to restore color and other-vector-valued images, we will discuss this total variation in section 4.3.

This norm can't be compared with others norms because it does not represent a local variation in an image. Finally, he proposes a natural choise for the function of Sapiro f, he chooses $f(.) = \sqrt{\Lambda_+ + \Lambda_-}$, i.e, the square root of the trace of the multi-spectral tensor.

4. SOME MODELS FOR COLOR IMAGE RESTORATION

In the scalar case, the anisotropic diffusion is based on the local variation of the gradient. For a color image, it is necessary to take into account vectorial information, i.e. the three color channels should not be restored independantly. Several methods for color image restoration has been developped. Among these methods, we find the diffusion models of Sapiro, Blomgren and Tchumperlé.

4.1. Diffusion of Sapiro:

Sapiro proposes following restoration model :

$$\frac{\partial I}{\partial t}(x, y) = g(N(x, y)) I_{\xi\xi} \tag{11}$$

Where $N = \sqrt{\Lambda_+ - \Lambda_-}$ is gradient norm of Sapiro, ξ is the direction of the weaker variation, and g is a decresing positive function such that :
$g(t) \dashrightarrow 1$ when $t(t) \dashrightarrow 0$
$g(t) \dashrightarrow 1$ when $t(t) \longrightarrow \infty$
$g(t)$ can be choosen as : $g(t) = \exp\left(-\frac{t}{k}\right)^2$.
We can notice that vectorial information comes both from the three RGB channel of the color image by $g(N(x, y))$ and ξ.
The diffusion is allways done in the direction ξ, near edges, the diffusion decreases.

4.2. Diffusion of Tchumperlé :

Tchumperlé proposes following PDE :

$$\begin{aligned} \tfrac{\partial I}{\partial t}(x, y) &= \alpha_a (I - I_0) \\ &+ \alpha_a (g_1(N(x, y)) I_{\eta\eta} + I_{\xi\xi}) \\ &- \alpha_r (1 - g_1(N(x, y)))\, sign(G \star I_{\eta\eta} I_\eta) \end{aligned} \tag{12}$$

Where :
$N(x, y) = \sqrt{\Lambda_+}$.
$g(t) \dashrightarrow 1$ where $t(t) \dashrightarrow 0$
$g(t) \dashrightarrow 0$ where $t(t) \longrightarrow \infty$
$g(t)$ can be choosen as : $g(t) = \exp\left(-\frac{t}{k}\right)^2$.

Here too, the vectorial information comes from the three RGB channels. Color information is taken handled in $g(N(x, y))$ and also in the two directions of diffusion ξ and η.
In the homogeneous areas, the diffusion is isotropic, but near edges the diffusion is done mainly in the direction ξ.

4.3. Diffusion of Blomgren

Blomgren proposes the use of the total variation norm for vector-valued images defined by the following :

$$TV_{n,m}(I) = [\Sigma_{i=1}^{m} TV_{n,1}(I^i)^2]^{1/2}$$

with:
$$TV_{n,1}(I) = \int_\omega \|\nabla I\| dx$$

and I^i is the i^{th} component of I and $(m = 3, n = 2)$.
We now consider the minimization of this expression. The corresponding Euler-Lagrange equation are :

$$\frac{\partial I^i}{\partial t}(x, y) = \frac{C^i}{[\beta + \|\nabla I^i\|]^{1/2}} I^i_{\xi\xi} \tag{13}$$

with $c^i = \frac{TV_{n,1}(I^i)}{TV_{n,m}(I^i)}$ and β is a small regularization parameter, introduced to avoid division by zero.
Vectorial information is taken into account only in the coeffient c^i.
In order make this diffusion scheme stable, the time step must be very small, and consequently, the number of iteration must be very high.

5. AN OPTIMAL PDE FOR COLOR IMAGE RESTORATION

Given a noisy color image I, We want the image to be smoothed in the direction parallel to object boundaries and prevent as much as posssible, diffusion across edges. Moreover want to control efficiently the diffusion near corners.

Suppose that an edge on the image I but he is not present on one or tow plan $\{R, G, B\}$ corresponding to color image I. In one hind, to control the diffusion near boundaries, it was not necessary to do it with used the norm of color gradient. On evry plan$\{R, G, B\}$, we can just use the norm of gradient associated to this plan (gaussian gradient for example). In the other hind, to control diffusion near corners, we use the color curvature computed in section 3.1(see equation 9).

5.1. Our diffusion PDE

We propose the equation of anisotropic diffusion for color image :

$$\begin{cases} \frac{\partial I^{i,\,n+1}}{\partial t} = g(\phi^n)\, I^{i,\,n}_{\eta\eta} + f(\mathcal{K})\, I^{i,\,n}_{\xi\xi} \\ \phi^n \quad = |\, \nabla I^{i,\,n} \star G(x, y)\, | \end{cases} \tag{14}$$

where $i \in \{R, G, B\}$ and N_i is the norm of gaussian gradient associate to evry composantes for the color image I and $G_t(x, y)$ is a Gaussian function :

$$G_t(x, y) = Ct^{-1} \exp\left(-\left(x^2 + y^2\right)/4t\right) \tag{15}$$

$\mathcal{K}$ is a curvature defined in (9), η and ξ are the tow directions of diffusion defined in (3.1), $f(.)$ and $g(.)$ tow nondecresing functions.

Color information is tanken into account only in the direction of diffusion ξ *and* η. It is clear that when $\|\nabla (G_t \star N_i)\| = 0$ with $N_i \in \{R, G, B\}$, the diffusion is optimale.

6. RESULTS OF COLOR IMAGE RESTORATION

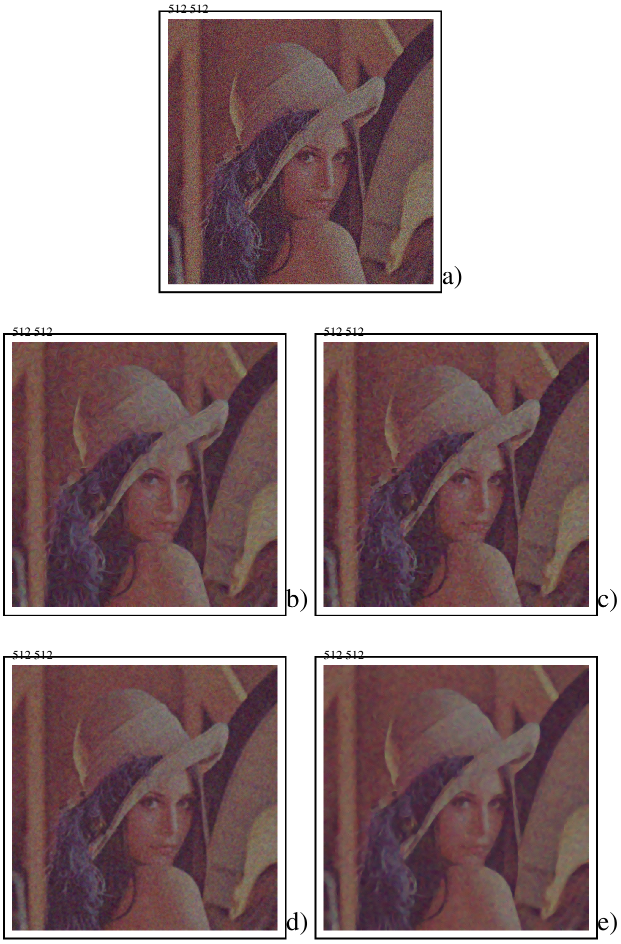

Fig. 2. *a) Image couleur réelle bruitée(100×100), b) Restoration of Blomgren 200 itérations and dt = 0.001, c) Restoration of Sapiro 20 itérations and dt = 0.1, d) Restoration of Deriche 20 itérations and dt = 0.1, e) Our model of restoration 50 itérations and dt = 0.1*

7. CONCLUSION

In this paper, we have presented a new PDE for color image restoration.A general framework of anisotropic diffusion of color images was presented.We have show here that our approach based on PDE take into account the pixel vectorial information whose taken by the two directions of diffusion η and ξ, and control well the diffusion near edges and corners.

8. REFERENCES

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