# Color Images De-Noising by Wavelet Maxima Representation and Regions Segmentation

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## Abstract

In this paper, we present a multiresolution approach for color image de-noising by image reconstruction. The proposed method relies on a modified wavelet reconstruction algorithm. This algorithm does not use the wavelet maxima generated by decomposition but a new maxima representation. This new representation is calculated from a multi-scale region segmentation approach. The multi-scale segmentation process allows us to obtain closed edges (vital for a correct image reconstruction) and a more simple selection of most significant edges (the less affected by noise). The image de-noising result is judged by the uniformity of the reconstructed image regions.

## Introduction

Approaches, using wavelet analysis as a framework, have achieved significant success for image noise reduction.[8,9] The image is decomposed using orthogonal or biorthogonal[11] filter banks, and the components are decimated[6,11] or undecimated.[12] Noise reduction may be performed on each component using correlation between different scales,[8] local Lipschitz regularity,[9] multiscale Kalman smoothing filter,[13] Bayesian hypothesis on the noise characteristics, thresholding of the components pixel values[11] or by an image reconstruction from wavelet maxima.[6] In this paper a technique for reduction of noise, using the wavelet reconstruction algorithm is presented. To decompose and reconstruct the image a filter bank derived from a biorthogonal set is utilized and the components are undecimated. The noise reduction is performed by reconstruction from a new wavelet maxima representation. This new representation is obtained by combining two classical image segmentation techniques: region segmentation and edges segmentation. Well-closed edges will be extracted form multi-scale region segmentation, avoiding in this way the well-known problem for edge detection: maxima linking procedure.

## Representation in Wavelet Maxima

The discrete wavelet decomposition associated to a particular base allows us to build a multi-scale gradient representation. To create this representation we choose, as S. Mallat suggested, a function $\psi$ which is the first derivative of a smoothing function $\phi$ (function which allows us to calculate the image at different resolutions). For the function $\psi$ to be an admissible wavelet function, it has to verify certain number of proprieties.[2] In the case of an image, we introduce two wavelet functions $\psi^1$ and $\psi^2$ as the first partial derivatives of function $\phi$:

$$\psi^1(x,y)=\frac{\partial\phi(x,y)}{\partial x} \text{ and } \psi^2(x,y)=\frac{\partial\phi(x,y)}{\partial y} \qquad (1)$$

For all functions $f\in L^2(\Re^2)$, the wavelet transform, at scale $l$, has two components:

$$d_l^1(x,y)=f*\psi_l^1(x,y) \text{ and } d_l^2(x,y)=f*\psi_l^2(x,y)$$

with

$$\psi_l^a(x,y)=\frac{1}{\sqrt{2^l}}\psi^a\left(\frac{x}{2^l},\frac{y}{2^l}\right)$$

From the linearity of convolution and derivation, it can be verified that:

$$\begin{bmatrix} d_l^1(x,y) \\ d_l^2(x,y) \end{bmatrix} = \sqrt{2^{-l}} \begin{bmatrix} \frac{\partial}{\partial x}(\phi_l * f)(x,y) \\ \frac{\partial}{\partial y}(\phi_l * f)(x,y) \end{bmatrix} \qquad (2)$$

Equation (2) indicates that the wavelet transform (with the wavelet functions defined by equation (1)) of an image at scale $l$ is equivalent to the gradient of the image $f$ smoothed by the function

$$\phi_l(x,y)=\frac{1}{\sqrt{2^l}}\phi\left(\frac{x}{2^l},\frac{y}{2^l}\right) \qquad (3)$$

Then, a maximum in this wavelet transform corresponds to an inflection point on the curve $\phi_s*f$.

The discrete wavelet transform thus defined is calculated thanks to the algorithm "à trous".[1] The decomposition of an image $f \in L^2(Z^2)$ at scale $l$ is calculated by convolution with two filters $h^l$ and $g^l$. $h^l$ and $g^l$ are obtained by inserting $l-1$ zeros between coefficients of $h$ and $g$, low pass and high pass filters associated to functions $\phi$ and $\psi$.[1] In order to obtain a perfect reconstruction, it is necessary and sufficient that a group of synthetic filters $q$, $p$ and $z$ exist which confirms the following condition[2] ($\hat{h}$ is the Fourier transform of the set $h$):

$$\hat{h}(w_x)\hat{p}(w_x)\hat{h}(w_y)\hat{p}(w_y)+\hat{g}(w_x)\hat{q}(w_x)\hat{z}(w_y)+ \\ +\hat{z}(w_x)\hat{g}(w_y)\hat{q}(w_y)=1 .$$

From wavelet coefficients

$$\begin{bmatrix} d_l^1(x,y) \\ d_l^2(x,y) \end{bmatrix}$$

a change in indicator is carried out by:

$$\rho_l = \|\nabla_l g\| = \sqrt{\left(d_l^1\right)^2 + \left(d_l^2\right)^2}$$

and

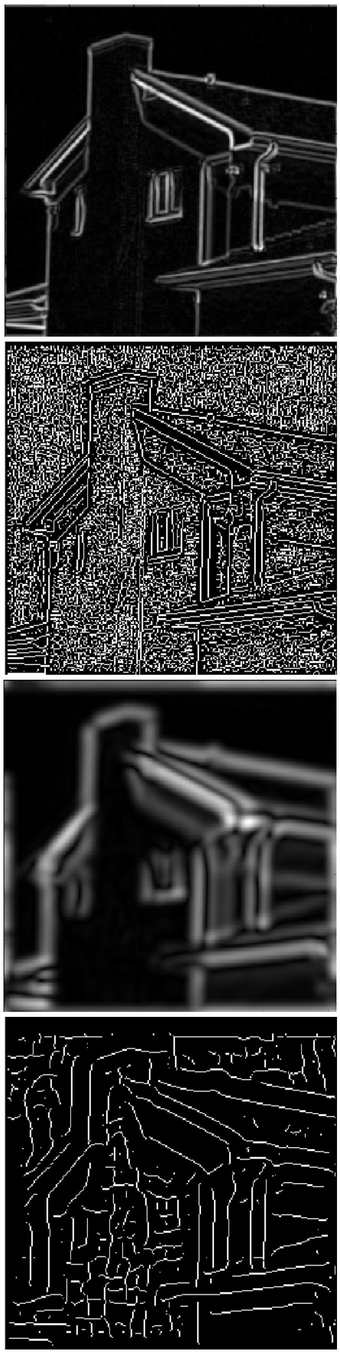
$$\theta_l = \arg(\nabla_l g) = \arctan\left(\frac{d_l^2}{d_l^1}\right) \qquad (5)$$

Repeating the Canny's formalism,[3] the maxima, or edges, at scale $l$ will be the local maxima points of the norm $p_l$ in the direction $\psi_l$. The set thus defined will constitute the representation in wavelet maxima of an image.[2] In the next figure we present an example for two scales of resolution (figure 1).

Mallat and Zhong have proposed an iterative algorithm for an image reconstruction, using this maxima representation.[6] The wavelet and scale functions used for 2D multi-scale decomposition are elements of modified spline family.[5] This decomposition was extended for color images, using a marginal approach, in Ref. 4.

## Edges Detection from a Region Segmented Image

Some methods proposed in the literature uses the multi-scale gradient for image compression and restoration. For example, the wavelet decomposition can be used to make a multi-scale selection of the most significant maxima, which corresponds to the most significant objects of the image. From that, an image compression or reconstruction is possible. However, one of the weaken points of those methods remains the maxima linking procedure over and between scales. The linking procedure is important because it determinates the quality of reconstructed image. All of those techniques detect the edges, according to Canny's criteria, for all scales from the gradient image. The resulting edges are not closed and very sensitive of noise (figure 1). In order to avoid this problem, we propose to extract the edges using a region approach at each scale. Thus we can obtain closed edges and a more simple selection of most significant edges.



a) b) c) d)

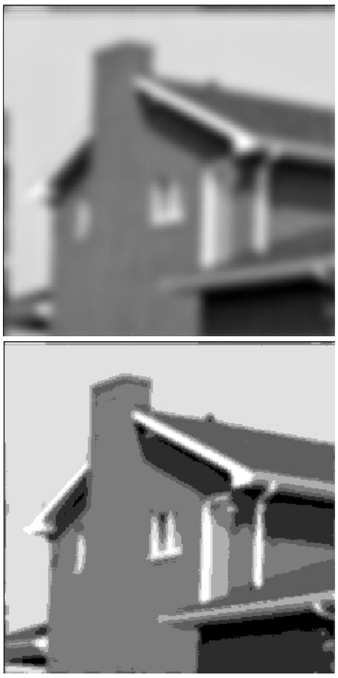
*Figure 1. a),c) the norm of the gradient at the second and fourth scale; b),d) the maxima at the second and fourth scale.*

The region segmentation method used in this paper is a thresholding technique.[7] For that we detect the histogram local maxima (followed by threshold step). For an image $f$ of size $M$ x $N$ pixels and $L$ levels of grey, the image histogram it's defined (6) as the occurrence probability of different grey levels.

$$h(i) = \frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\delta\big(i - f(m,n)\big),\ i=0,1,\ldots,L-1 \qquad (6)$$

The number of those maxima represents the number of classes. The limits of each class are defined as the median grey level between the current maxima and his predecessor and the median grey level between the current maxima and the next one. After classification, the edges are extracted from the borders of those different regions.

We attempt to extract the edges at different scales. For that we compute different image approximations at different scales (thanks to a low pass filtering, with $h^l$ filter presented in the first paragraph). Then, over those images we apply our region segmentation algorithm.



a)

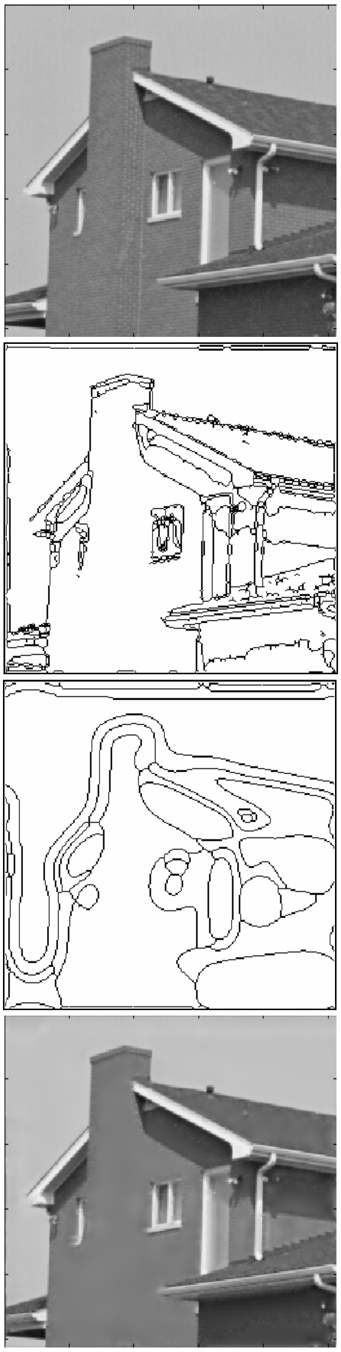
b)

*Figure 2. image approximation at the second scale; b) image a) segmented by regions.*

Thus, we make an association between the edges extracted by the region approach and the gradient values for different scales, and we obtain a new wavelet maxima representation, for which only the most significant edges are detected and they are closed. In those conditions we can apply the reconstruction algorithm proposed by Mallat.

We can also adapt this approach for the color images. The region segmentation method used in this paper is based on a technique named "*bits interlacing*".[10] This one allows us to transform the tri-dimensional representation of a color image in a scalar representation. So we can apply the segmentation method used for the grey level images. As we have said, the color gradient representation is obtained with a marginal approach. The next figure illustrates an example of reconstruction of a color image from the new edge representation (figure 1).

As we can see from those images, the reconstructed image is very similar to the original image, and the correspondence between the colours of the two images is an excellent one.



a)

b)

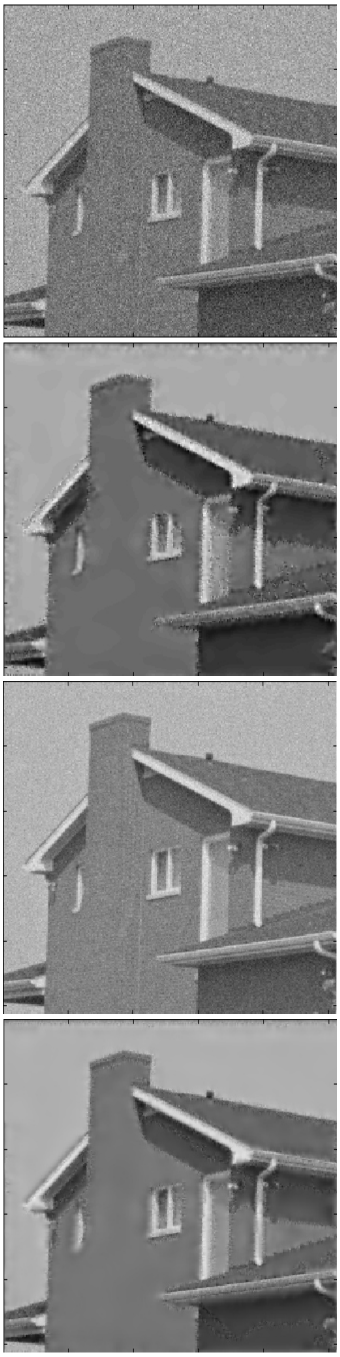
c)

d)

*Figure 1. a) Color image; b) Edge image at second scale; c) Edge image at fourth scale; d) Reconstructed image.*

## Applications for Noisy Images

Image de-noising was one of the first application domain for the wavelet maxima.[8,9] In this context, we attempt to preserve only the coefficients corresponding to the information and to eliminate those generated by the noise. Thanks to its classes uniformity criteria, the region segmentation gives us already an initial de-noised image. However, in the case of a segmented noisy image, the presence of noise is translated into small size regions, which we must eliminate. This elimination is realised using a minimal surface criteria. This is made for each image approximation at all scales of the wavelet decomposition. The algorithm used to fill those regions is based on a mathematics morphology technique named "*region filling*".[7]



*a)* *b)* *c)* *d)*

*Figure 2. a) Noisy gray level image ; b) Reconstructed image ; c) Noisy color image ; d) Reconstructed image.*

The edges extracted from this new image are the most significant edges and they are well linked. In order to improve the quality of those edges we apply another technique of mathematics morphology, named "*morphological skeleton*".[7] With this new set of edges and the iterative reconstruction algorithm, we obtain an image that is very closed to the original one, but de-noised. The next figure presents an example of reconstruction, de-noising, of a grey level image and a color image (figure 2).

As we can see, the two noisy images are correctly reconstructed and the correspondence between colours is excellent. The uniformity of the reconstructed image regions allows us to consider that, by this reconstruction, we have made an image de-noising.

## Conclusion

We have presented a noise reduction technique for color images, based on the Mallat image reconstruction algorithm, using a new wavelet maxima representation. This new maxima representation allows us to compute a multi-scale noise reduction algorithm.

The experiments have proved that the proposed technique, offers good result both for the grey levels images and color images. The reconstruction quality is jugged using a human psycho-visual evaluation. A mathematically evaluation is also possible using for example a PSNR (peak signal-noise ratio), but the calculated value of PSNR is not very well adapted for the case of color images, the result being very approximately.

Future improvements can be made to this work, for example by using a more efficient region segmentation technique, both for grey level images and color images.

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## Biography

Adrian Stoica received his B.S. degree in Electronics and Telecommunications from the Polytechnic University of Bucharest in 2000 and a Master in Transmission of information: Informatics, Image and Automatic from University of Poitiers in 2001. Actually he is a Ph.D student at the University of Poitiers and Polytechnic University of Bucharest. His work focused on the optimisation of the new image compression standard JPEG 2000.