A Mathematical Analysis of Coarseness of Color Variation in Painting Arts

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Abstract

Coarseness or fineness of spatial color variation is one of the valuable features of a color image.

This paper proposes a new metric for measurement of intensity of spatial color variation by mean of Haar wavelet transform.

Several paintings are analysed by our method. The characteristics of the paintings are well illustrated by the result of the analysis.

1. Introduction

The pointillism, one of the painting techniques typically used in the works of G. Seurat, has two characteristic variation of colors. One is minute color variation by small dots in paint. The other is broad color variation entailed in the composition of an image.

These kinds of color variations are caused by color differences between juxtaposed colors. This paper proposes a new measure on the coarseness (or fineness) of the placement of color differences on a picture plane, which characterizes color features of an image.

In order to make a measure, one may think of an application of Fourier expansion. However, convergence of Fourier coefficients is slow when an image has discontinuity of color on the picture plane. For this reason, there might be a difficulty to interpret Fourier coeffecients, especially for high frequency.

Alternatively we may use an expansion with piecewise continuous bases, e.g. Walsh transform. In this case, some bases have double periods, and the meaning of coefficients for a double periodic basis is not clear.

It follows from what has been said that Haar wavelet transform become a candidate. We devised a new comprehensive measure composed of wavelet coefficients, which represents the magnitude of color difference for each wave-length and has good property for discontinuity of color.

2. Theory

2.1. A Wave-Length Analysis for Spatial Color Variation

First consider one dimensional case for simplicity. Suppose a picture plane is an interval [0, 1] and an image is expressed by a piecewise continuous real function f on [0, 1].

Divide the interval [0, 1] into $M = 2^{m-1}$ subintervals of width $h_m = 1/M = 2^{-m+1}$. Bisecting each subinterval into two juxtaposed resions, define a local color difference between these two resions by

$$\Delta f_i^m = f^m(x_i)\ \ f^m\left(x_i + \frac{h_m}{2}\right) \qquad (1)$$

where

$$\overline{f}^m(x) = \frac{1}{h_m/2}\int_x^{x+h_m/2} f(\xi)d\xi \qquad (2)$$

is an average color in the interval $[x, x + h_m/2]$ and $x_i = ih_m(i = 0,1,\ldots M)$ is a equi-distant point on [0,1].

An average color difference can then be calculated as a root mean square of all local color differences:

$$\overline{\Delta E}^m = \left\{\frac{1}{M}\sum_{i=0}^{M-1}\left(\Delta f_i^m\right)^2\right\}^{1/2} \qquad (3)$$

The average color difference $\overline{\Delta E}^m$ represents an intensity of spatial color variation for a given wave-length h_m.

Now consider the general case. Let an image f be a piecewise countinuous mapping from a square domain $[0, 1]^2$ (a square picture plane) to three-dimensional Euclidean space R^3 (a color space).

Let $M = 2^{m-1}$ and $N = 2^{n-1}$ be respectively a number of horizontal division and a number of vertical division of the domain $[0, 1]^2$. Put $x_i = ih_m(i = 0,1,\ldots,M)$ and $y_j = jh_n(j = 0,1,\ldots,N)$.

Applying the above one-dimensional definition (1) to x axis and then to y axis successivly, a local color difference $\Delta f_{i,j}^{m,n}$ can be defined on each sub-domain $[x_i, x_{i+1}]$ x $[y_j, y_{j+1}]$. Note that the local color difference $\Delta f_{i,j}^{m,n}$ is a vector in R^3 which corresponds to three components of a color.

The average color difference with a horizontal wave-length h_m and a vertical wave-length h_n is calculated by the equation

$$\overline{\Delta E}^{m,n} = \left\{\frac{1}{MN}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}\left\|\Delta f_{i,j}^{m,n}\right\|^2\right\} \qquad (4)$$

where $\|\cdot\|$ denotes an Euclidean norm in R^3.

$\overline{\Delta E}^{m,n}$ represents an intensity of spatial color variation with a wave-length pair (h_m, h_n). (Note that $\overline{\Delta E}^{m,n}$ is a scaler.)

The case that an image is not square, suppose that the height is less than the width and the width is equal to 1, without loss of generality. In this case change N in the equation (4) to

$$N' = [h \cdot N],$$

where $h(< 1)$ is the height of a picture plane.

The sub-domain for calculating the local color difference is contained in the picture plane either entirely or partially. For the former, calculate the local average color by the definition itself. For the latter, calculate the local average color for a part of the sub-domain contained in the picture plane.

The algorithm to obtain an intensity matrix of spatial color variation from an original image is illustrated in Fig. 1.

2.2. Relation Between an Average Color Difference and Haar Wavelet Coefficients

For $m = 1, 2, \ldots$, and $i = 0, \ldots 2^{m-1} - 1$, Haar basis $\{\varphi_{m,I}\}$ is a complete orthnormal basis in $L^2[0, 1]$ and can be defined by

$$\varphi_{m.i}(x) = \begin{cases} 2^{(m-1)/2} & (x_i \le x \le x_i + h_m/2), \\ -2^{(m-1)/2} & (x_i + h_m/2 \le x \le x_{i+1}), \\ 0 & \text{other}, \end{cases}$$

$$\varphi_{m.i}(x) = \begin{cases} 1 & (0 \le x \le 1), \\ 0 & \text{other} \end{cases}$$

where $h_m = 1/2^{m-1}$ and ih_m.

In one-dimensional case, an expansion coefficient of Haar wavelet transform of an image $f \in L^2[0, 1]$ is given by $< f, \varphi_{m,i} >$, where $< \cdot, \cdot >$ represents an ordinary inner product in $L^2[0, 1]$. The following relation between an expansion coefficient $<f, \varphi_{m,I}>$ and a local color difference Δf_i^m holds:

$$\Delta f_i^m = -2^{(m-1)/2} < f, \varphi_{m,i} > .$$

Using (3) and noting that $M = 2^{m-1}$, it is found that the square of the average color difference is represented original image by the squared sum of expansion coefficients ("Power spectrum")

In the general case

$$\Delta f_{i,j}^{m,n} = 2^{(m-1)2+(n-1)/2} << f, \varphi_{m,i} >, \varphi_{n,j} >$$

which follows that the square of average color difference equals to the squared sum of expansion coefficients of two-dimensional Haar wavelet transform:

$$\left(\overline{\Delta E}^{m,n}\right)^2 = \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} \left\| << f, \varphi_{m,i} >, \varphi_{n,j} > \right\|^2.$$

These relations tell that a color image can be restored from a set of local color differences and results of color image analysis using Haar wavelet transform can be interpreted in terms of color difference.

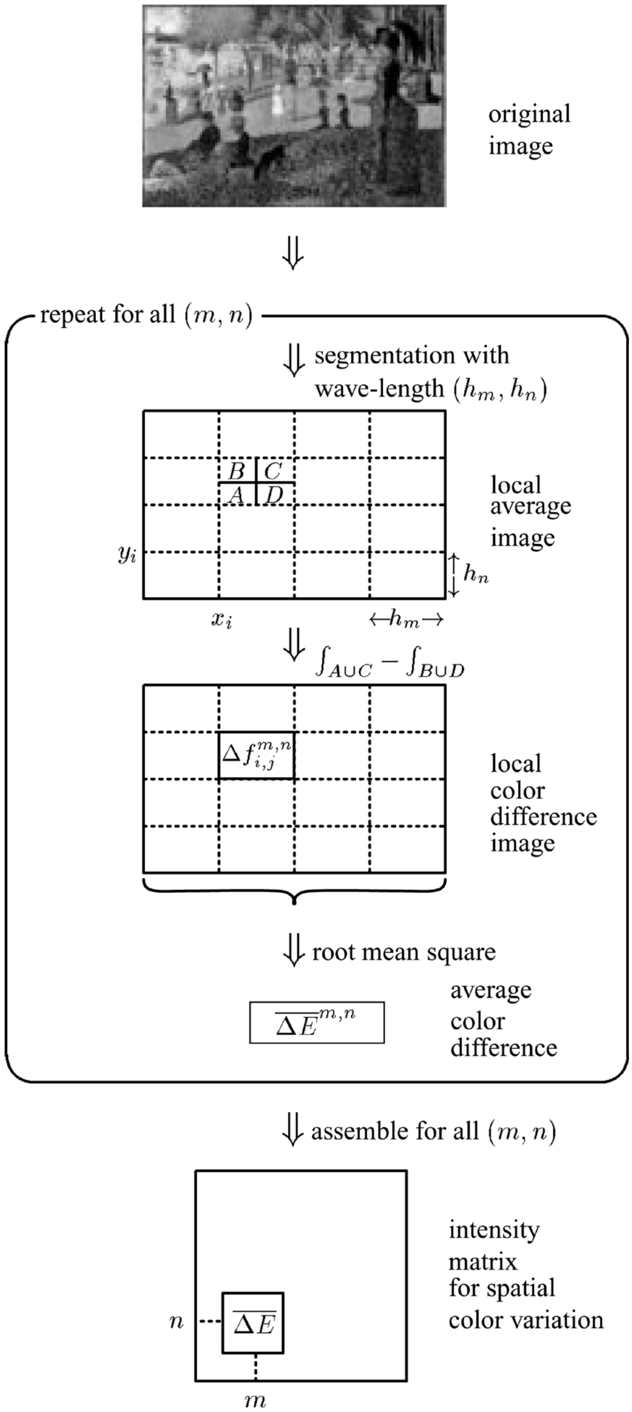

Figure 1. Algorithm to obtain an intensity matrix

3. Application

3.1. Experiment in CIELUV

The above wave-length analysis was applied to some number of paintings (about 200 works of 11 artists).

Most of the source images are taken from art books or catalogues of exibitions and scanned by an image scanner. A few images are directly obtained by a digital still camera. Their color values are originally represented by RGB, which are transformed to CIEXYZ, then converted to a uniform color space CIELUV.

A color value in CIELUV consists of three components L^*, u^*, v^*, from which an auxiliary attribute $C^* = \sqrt{u^{*2} + v^{*2}}$ are computed. In this experiment, a local color difference image and an average color difference were calculted for each component and for each color difference component. A computation flow diagram to obtain intensity matrices in CIELUV is given in Fig. 2.

Four images of color components L^*, u^*, v^* and C^* are prepared first. For a given pair (m,n), divide each image into M x N segments (M, N are already defined in the previous section) to make a corresponding local color difference image (through a local average color image). From local color difference images ΔL, Δu, Δv, and ΔC, two more local color difference images ΔE and ΔH are calculated, using the relation $\Delta E^2 = \Delta L^2 + \Delta u^2 + \Delta v^2$ and $\Delta H^2 = \Delta E^2 - \Delta L^2 - \Delta C^2$ (here superscript (m,n) and subscript (i,j) are omitted for readability). Then take root mean squares of all local color differences to obtain the average color differences $\overline{\Delta L}^{m,n}$, $\overline{\Delta u}^{m,n}$, $\overline{\Delta v}^{m,n}$, $\overline{\Delta C}^{m,n}$, $\overline{\Delta E}^{m,n}$ and $\overline{\Delta H}^{m,n}$.

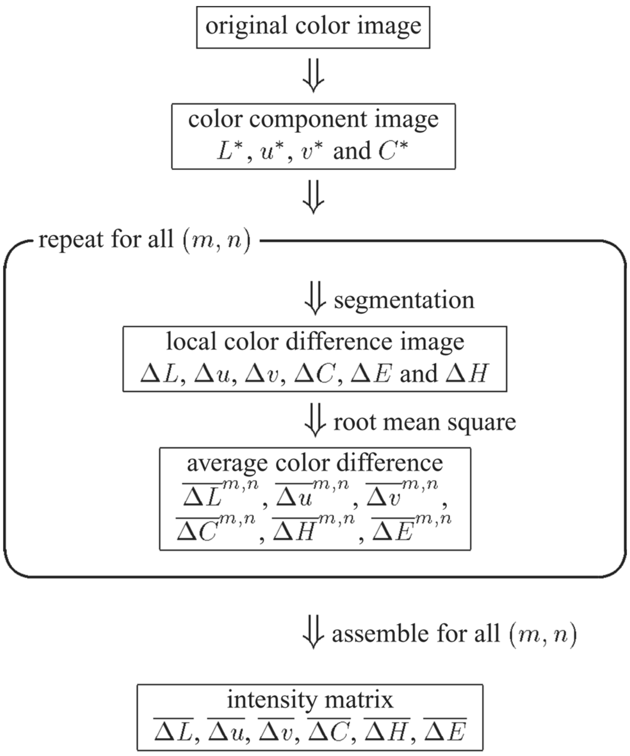

Figure 2. Computation flow in CIELUV

Lastly assemble average color differences for all (m,n) (m and n are restricted to be finite) to make intensity matrices of spatial color variation $\overline{\Delta L}$, $\overline{\Delta u}$, $\overline{\Delta v}$, $\overline{\Delta C}$, $\overline{\Delta H}$, and $\overline{\Delta E}$.

An intensity matrix of spatial color variations is interpreted as follows (Fig. 3). The larger is the parameter m, the finer is the horizontal color variation. The larger is the parameter n, the finer is the vertical color variation.

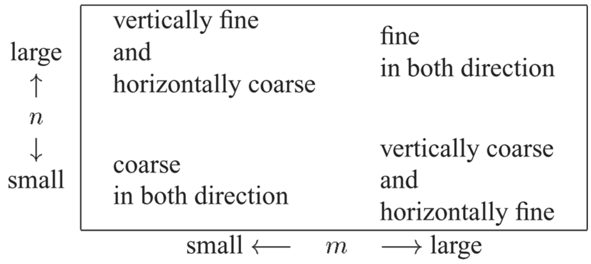

Figure 3. Meaning of an intensity matrix of spatial color variation

The column $m = 0$, $n \geq 1$ shows the spatial color variation of average color for the horizontal direction and the row $n = 0$, $m \geq 1$ shows the spatial color variation of average color for the vertical direction.

3.2. Results

Four results of the experiment are shown in Fig. 4, which illustrate typical color variation from fine touch to coarse composition. Two-digit number in the entry (m,n) of an intensity matrix indicates the degree of intensity of spatial color variation with a wave-length pair (h_m, h_n).

(a) V. Gogh, "Starry Night", 1889

Comparison of three intensity matrices of spatial color variation $\overline{\Delta L}$, $\overline{\Delta u}$, and $\overline{\Delta v}$ clearly indicates $\overline{\Delta v}$ is dominant. This means, the color changing along v axis ("blue - yellow" direction) is stronger than the others. This might be caused by the contrast of yellow stars and the crescent and bluish night sky.

The swirled starry sky accross the canvass brings large value of $m = 0$, $n = 2$ in $\overline{\Delta v}$. The bright moon and twinkling stars express large color difference in lower-left region of $\overline{\Delta v}$. Slightly large values in the upper-right region of $\overline{\Delta v}$ are caused by the painter's strong touch with blue, yellow, and orange colors.

(b) P. Picasso, "Mother and Child", 1921

The painting is mainly composed of the gradation of gray and skin color. In $\overline{\Delta L}$ and $\overline{\Delta C}$, the gradation appears as continuative change of 'wave-length' component. Comparing $\overline{\Delta L}$, $\overline{\Delta C}$, and $\overline{\Delta H}$, the values of $\overline{\Delta L}$ are slightly greater than $\overline{\Delta C}$ and the values of $\overline{\Delta H}$ are very small. This tells that the painter uses gradation in lightness and in chroma, not in hue. In $\overline{\Delta H}$, short 'wave-length' components are decreasing rapidly, which suggests discontinuity of hue change.

Roughly speaking the painter uses only two colors, gray and skin color, and their gradation.

(c) P. Klee, "Highway and Byways", 1929

The painting is composed of many horizontally oblong rectangles whose colors are orange, yellow, blue, grayish green and greyish purple. This leads to the large values in the upper-left region of the intensity matrix. The values of $\overline{\Delta C}$ dominate the others, because the chroma of orange or yellow is quite higher than the other colors.

(d) G. Seurat, "A Sunday Afternoon on the Island of La Grande Jatte", 1884-86

Comparing three intensity matrices $\overline{\Delta L}$, $\overline{\Delta C}$, and $\overline{\Delta H}$, it is found that, for any entry of the matrices, the value of $\overline{\Delta C}$ is greater than the other two, $\overline{\Delta L}$ and $\overline{\Delta H}$. This means that chroma variation is an effective and dominant technique in the painting. Especially, the upper-right region of $\overline{\Delta C}$ has large values, which is caused by fine touch of brush, i.e., 'pointillage'.

The reason that the values of $\overline{\Delta H}$ are small is presumed by the fact that the most part of the canvass except persons is painted mainly in green of the same hue. As for lightness the canvass is divided into two parts, sunny upper half and shadowy lower half. This characteristic light-dark composition is shown in the entry $m = 0$, $n = 2$ in $\overline{\Delta L}$.

4. Conclusion

A new method based on Haar wavelet transform to measure the magnitude of spatial color variation for each wave-length was proposed. This method has good property for discontinuity of color on a picture plane. The mesure used in the method is not only a simple mathematical quantity but also a percentual metric, which has a deep connection with color difference generated at the border of juxtaposed colors.

Several paintings are analysed using our method, and the results well correspond to our intuitive impression on color variation such as fine touch of brush and broad arrangement of colored regions.

There might be many applications of our method, one of which is a field of image retrieval by contents.

5. References

1. G. G. Walter, Wavelets and other orthogonal systems with applications, C.R.C Press, Inc., 1994.

6. Biography

KOBAYASI Mituo received his Ph.D. in Engieering from the University of Tokyo. He is a professor at the University of Electrocommunications in Tokyo and a visiting professor at National Museum of Japanese History. Mathematical research in color science, development of color infomation processing system, and computational analysis of color aesthetics are his major subjects of studies. He is a director of the Color Science Association of Japan and a member of the Japan Society for Industrial and applied Mathematics.

MUROYA Taizo obtained his master's degree in Engineering from the University of Electro-communications in Tokyo in 1993. Since 1995 he has joined in the National Museum of Modern Art, Tokyo to construct and manage of infomation systems. At present he belongs to the University of Electro-Communications as an research fellow. His research interests include mathematical problem in color imaging science. He is a member of the Color Science Association of Japan and the Japan Society for Industrial and applied Mathematics.