Spectral Sharpening: What is it and why is it important?

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Abstract

XYZ functions and cone sensitivities appear to play little role in visual perception in that colour computation does not appear to be carried out in cone or XYZ coordinates. In its first incarnation, spectral sharpening was proposed as a method for finding the color space, a linear combination of the cones, that best supported adaptation by a von Kries type model. The term sharpening is used because the resultant sensitivities have narrower support compared with the cones. In this paper we show that spectral sharpening also helps us to understand metamerism and color matching.

1. Introduction

Colour Science is a well established discipline with many well tested rules and assumptions. As such, research in colour often addresses detailed questions. For example, we might ask "how many sample points do we need to measure for the purposes of colour reproduction?" as Spiekermann et al did[18] or carry out detailed data analysis which addresses the question of "how well given chromatic adaptation formulae accounts for a given set of observer assessments?" e.g. see[13]. These are, of course, interesting questions and it is a worthy endeavour to search for their answers. However, perhaps there is scope to revisit the most basic assumptions used in colour science to see if we might find different ways for carrying out colour computation? This idea should not be considered as controversial. As an example, we all know that the standard XYZ colour space has little to do with human visual perception. Rather it was designed (in part) to make pencil and paper calculations easy. Given a more in depth understanding of visual processing and the tools of modern mathematics it is interesting to ask whether a different colour space might make color computation easier?

To answer this question we will look at three of the most fundamental topics of study in colour science: *Metamerism*, *Colour matching* and *Colour Adaptation*. I will argue that in each case a single *Spectrally sharp* colour space can be usefully applied. In its simplest guise, Spectral sharpening is a method for finding the linear combinations of the cones that are maximally sensitive in the short-, medium- and long-wave parts of the visible spectrum.

Of course if we consider any possible sensor (and not just linear combinations of the cones), the sharpest sensors are sensitive to a single wavelength of light. While it is impossible to find linear combinations of the cones that are even close to having monochromatic sensitivity, it is useful to set out to attain this goal. In doing so the strong link between metamerism, adaptation and matching will be made clear. Moreover, we will argue later that the eye does in fact behave as if it had single wavelength sensitivities. So, for the rest of this introduction we will consider how we might measure colour at 3 wavelengths.

Let us consider cone response, single wavelength response (and ultimately spectral sharpening) from a mathematical perspective. Denoting the spectral power distribution entering the eye as $C(\lambda)$ and the 3 cone sensitivities as $L(\lambda)$, $M(\lambda)$ and $S(\lambda)$, the l, m and s cone response can be written as:

$$\begin{array}{rcl} l & = & \int_\omega L(\lambda)C(\lambda)d\lambda \\ m & = & \int_\omega M(\lambda)C(\lambda)d\lambda \\ s & = & \int_\omega S(\lambda)C(\lambda)d\lambda \end{array} \quad (1)$$

where ω denotes the visible spectrum (approximately 400 to 700 Nanometres).

Equation (1) can, from a computational perspective, be viewed positively and negatively. On the positive side the spectral functions—the spectral sensitivity of the eye and the light entering the eye— are 'coded' by three quantities: the scalars l, m and s. That is, colour vision is inherently 3-dimensional and not spectral in nature. From an information processing perspective this is an advantage: the more compact the visual encoding the more readily visual information might be processed. A compact representation has similar benefits for practitioners in colour science. The negative, however, is that to determine the cone responses

we need to full spectral measurements both of the cone sensitivities and the colour signal spectra entering the eye.

To get around the need to deal with spectra we might propose the following:

$$\begin{array}{rcl} l_i & \approx & L(\lambda_i)C(\lambda_i) \\ m_j & \approx & M(\lambda_j)C(\lambda_j) \\ s_k & \approx & S(\lambda_k)C(\lambda_k) \end{array} \quad (2)$$

where λ_i and λ_j and λ_k are three wavelengths. Of course, the measurements we might make at a single wavelengths are probably not the same as the integral responses in Equation (1) (the approximation could be quite poor. So, let us remove (for now) the cones from consideration and assume three sensors R_1, R_2 and R_3 that have zero sensitivity everywhere except at wavelengths i, j and k where $R_1(\lambda_i) = 1$, $R_2(\lambda_j) = 1$ and $R_3(\lambda_k) = 1$. Substituting these three sensors for the cones in Equation (2):

$$\begin{array}{rcl} r_1 & = & C(\lambda_i) \\ r_2 & = & C(\lambda_j) \\ r_3 & = & C(\lambda_k) \end{array} \quad (3)$$

Equation (3) is simpler than (1) in that there are now no spectral functions whatsoever. But, unfortunately the measurements r_1, r_2 and r_3 are not the same as l, m and s (to remove spectra from the equations we have removed removed the link to human vision). Yet, the whole of colour science is based on the premise of accurate knowledge of l, m and s (or equivalently, x, y and z). That Equation (3) does not yield cone responses is, however, not a problem so long as for a given r_1, r_2 and r_3 we can find the corresponding l, m and s. Indeed, we will show later that:

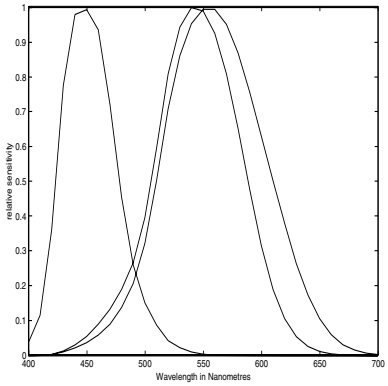
$$\underline{l} \approx \mathcal{T}\underline{r}_{i,j,k} \quad (4)$$

where $\underline{l}$ and $\underline{r}_{i,j,k}$ denote the three responses calculated in Equations (1) and (3) and $\mathcal{T}$ denotes a 3x3 matrix. The subscript $_{i,j,k}$ draws attention to the fact that measurements are only made at λ_i. λ_j and λ_k. Underscoring denotes vector quantities.

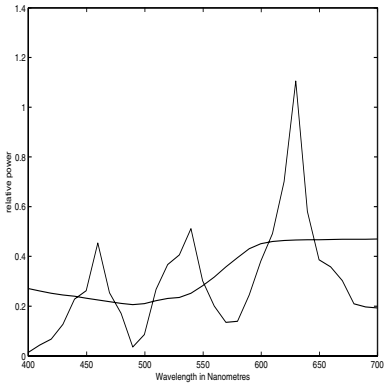
For the rest of this paper we will consider Equations (3) and (4) in the context of metamerism, adaptation and colour matching. In each case we will see that sampling spectra at three wavelengths leads to considerable computational simplicity. Moreover, in all cases the same three wavelengths turn out to be important. Finding linear combinations of cones that at these 'prime' wavelengths[1] yields sensitivities which are spectrally sharp. These sharp sensors provide a unified framework for understanding metamerism, adaptation and colour processing.

2. Metamerism and Measuring Colour

The trichromacy of the eye, the fact that spectral distributions entering the eye are coded by three numbers, is the key reason why colour reproduction works. Indeed, because spectral quantities 'project' to just 3 measurements it follows that there are many spectra that induce the same response. For a given observer and viewing condition, two spectra that look identical to an observer are called *Metamers*. In Figure (1a) the cone responses of the eye are shown. In Figure (1b) two metamers are shown (these metamers integrate to the same l, m and s response. The smooth curve is typical of the kind of spectra that occurs in nature, the jagged curve is indicative of the match made on a CRT.



1a. The cone Sensitivities



1b. Both spectra integrate to the same cone response

Suppose now we consider Equation (4) in more detail i.e. we would like to measure real spectra at just 3 wavelengths such that the resulting measurements are a linear transform from cone responses. How do we find the most appropriate wavelengths? One way to answer this question is to find the wavelengths i, j and k which minimises:

$$\min_{i,j,k} ||\underline{l}^{e,s} - \mathcal{T}_{i,j,k}\underline{r}^{e,s}_{i,j,k}|| \quad (5)$$

where e and s denote the eth illuminant and sth surface drawn from sets of measured surfaces. The 3×3 matrix $\mathcal{T}_{i,j,k}$ maps measurements in a least-squares sense at wavelengths i, j and k to corresponding cone responses. The

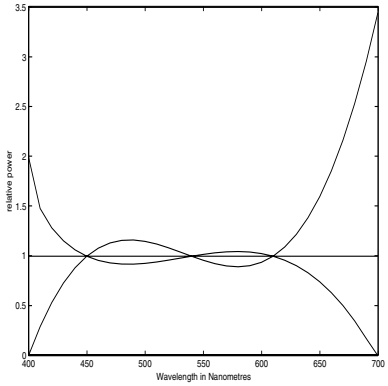
quantity that is minimises ($||.||$) could be RMS error or one weighted to take account of our perceptual response[17].

An optimisation of the form (5) was carried out in[5]. The 426 Munsell spectra[15] were used for exemplar reflectances and these were multiplied by 8 exemplar illuminants: 5 Judd[10] daylight phases (D48, D55, D65, D75 and D100) and CIE standard illuminants A, B and C. Minimising Equation (5), it was found that the optimal i, j and k wavelengths were: 450nm, 540nm and 610nm.

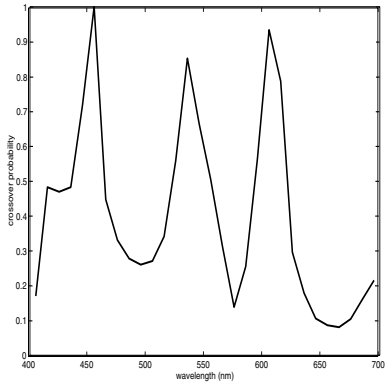
To assess colorimetric performance CIE Lab error was calculated. Under highly controlled viewing conditions a CIE Lab error of 1 correlates with a single just noticeable difference. In complex imagery an error of up to 3 is not significant[21]. For the Munsell dataset it was found that Equation (4) delivered an error of less than 2 CIE Lab units for 90% of the spectra. The 99th quantile error for the data set was just 8 CIE Lab units. We might reasonably conclude that, for the dataset tested, we can use Equation (4) instead of (1) without the loss of any colour accuracy.

However, the reader might feel uneasy that the 3 wavelength model is justified on what is a reasonably small set of measured spectra. Thus, we might look to other arguments to help determine which three wavelengths to use in Equation (4). Let us examine in more detail the shape of spectra that are metamers. Suppose that we measure a spectrum at wavelength i. We now change the spectrum but keep the power at i constant. Clearly the two measurements are the same at wavelength i and so are metamers (for a single sensor system that is sensitive only to this wavelength). In the general case where we measure light at wavelengths i, j and k, metamers are defined as having equal power at each of these three wavelengths: the spectral power at in between wavelengths is of no consequence. In Figure 2a we plot 3 metamers assuming measurements are made at 450nm, 540nm and 610nm. Notice the 3 spectra are equal at 450nm, 540nm and 610nm but are quite different elsewhere.

If metamers for cone sensors exhibited this same trend (metamers converged at wavelengths 450nm, 540nm and 610nm) then we might expect Equation (4) to work quite well. It has been shown that metamers for the human visual system must cross in at least 3 places across the visible spectrum[24]. In Figure 2b we show (for an infinite cardinality reflectance set [extrapolated from the set[23]] under a D65[10] Judd Daylight), the relative likelihood that metamers cross at particular wavelengths[6]. It is clear the most likely (by far) crossover wavelengths are again around 450, 640 and 610nm. The same wavelengths are found for different reflectance data sets and different viewing illuminants[6].



2a. Three metamers for sensors sensitive to 450nm, 540nm and 610nm



2b. The likelihood that metamers cross at particular wavelengths

3. Adaptation: mapping colours between illuminants

So far, we have not considered colour response across illumination. We have used the notation $C(\lambda)$ to denote the light entering the eye without making clear the role of light and surface. Here, let us denote the spectral power distribution of light as $E(\lambda)$ and surface reflectance $S(\lambda)$. The colour signal spectrum entering the eye is defined as:

$$C(\lambda) = E(\lambda)S(\lambda) \tag{6}$$

Substituting (6) into (3) and using the superscript ^{E} to denote dependence on a particular illuminant:

$$\begin{array}{rcl} r_1^E & = & E(\lambda_i)S(\lambda_i) \\ r_2^E & = & E(\lambda_j)S(\lambda_j) \\ r_3^E & = & E(\lambda_k)S(\lambda_k) \end{array} \tag{7}$$

Often we are interested in mapping responses captured with respect to one illuminant to a second illuminant. For example, if a picture is captured with a digital camera under indoor Tungsten light then, because Tungsten is yellowish, all the captured responses will have significant power

in the middle and longer wavelengths. By removing the relatively strong power in the longer wavelengths we can 'balance' the camera measurements (in terms of our example we might boost the short-wave measurement relative to the medium and longer wave measurements). Such balancing is a prerequisite for colour reproduction where the viewing illuminant (effective white point) will rarely be the same as the capture light. This balancing can be thought of as changing the illuminant (in this case from yellowish to whitish). In turn, mapping responses to white lighting conditions can be thought of as adapting to the illuminant since post-mapping any biases due to the illuminant colour will be attenuated.

According to (7) it is easy to work out how to balance colours relative to different light sources. Denoting response to $S(\lambda)$ under a second light E' as $\underline{r}^{E'}$, it is straightforward to show that:

$$\begin{aligned} r_1^E &= \frac{E(\lambda_i)}{E'(\lambda_i)} r_1^{E'} \\ r_2^E &= \frac{E(\lambda_j)}{E'(\lambda_j)} r_2^{E'} \\ r_3^E &= \frac{E(\lambda_k)}{E'(\lambda_k)} r_3^{E'} \end{aligned} \tag{8}$$

Rewriting (8) in matrix notation:

$$\underline{r}^E = \mathcal{D}^{E,E'} \underline{r}^{E'} \tag{9}$$

Where $\mathcal{D}^{E,E'}$ is a 3×3 diagonal matrix with diagonal entries equal to the ratios of the powers of E over E' for wavelengths i, j and k. Adaptation models of the form (9) (where the scaling factors are the ratios of illuminant responses) are sometimes called 'von Kries' type models.

Let us now re-examine Equation (4). If cone responses are linearly related to the measurements at three wavelengths then the converse is generally true:

$$\underline{l} \approx \mathcal{T} \underline{r}_{i,j,k} \Rightarrow \underline{r}_{i,j,k} \approx \mathcal{T}^{-1} \underline{l} \tag{10}$$

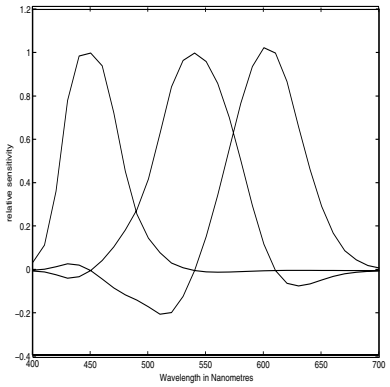
Substituting (10) into (9) we see that:

$$\mathcal{T}^{-1} \underline{l}^E \approx \mathcal{D}^{E,E'} \mathcal{T}^{-1} \underline{l}^{E'} \tag{11}$$

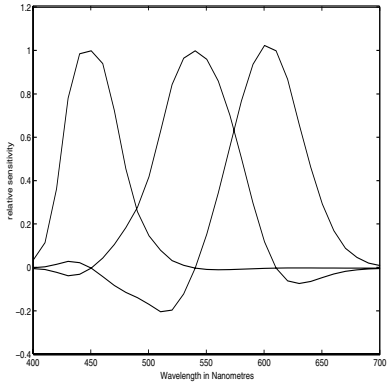
Pre-multiplying both sides by $\mathcal{T}$:

$$\underline{l}^E \approx \mathcal{T} \mathcal{D}^{E,E'} \mathcal{T}^{-1} \underline{l}^{E'} \tag{12}$$

Equation (12) informs us that cone responses across illumination are related by a set of 3 simple scalars. However, the scalars are not applied to the responses themselves but rather to the linear transform $\mathcal{T}^{-1}$ of the cone responses. In Figure 3a I have plotted the linear transform of the cones where $\mathcal{T}^{-1}$ is calculated using Equation (10) to find wavelengths i, j and k and Equation (4) to fix the transform. The calculation takes place under the maximum ignorance assumption[22] (that is, we assume all possible spectra might occur not just those in a training set). With respect to the maximum ignorance assumptions it can be shown that:



3a. Sharp sensors which behave like monochromatic sensors at 450nm, 540nm and 610nm



3b. Matching functions for primaries at 450nm, 540nm and 610nm

$$\mathcal{T} = \begin{bmatrix} L(\lambda_i) & L(\lambda_j) & L(\lambda_k) \\ M(\lambda_i) & M(\lambda_j) & M(\lambda_k) \\ S(\lambda_i) & S(\lambda_j) & S(\lambda_k) \end{bmatrix} \tag{13}$$

Looking at Figure 3a it is clear the sensors are sharper than the cones. This is not surprising: these sensors generate numbers that best approximate spectral measurements made at the wavelengths 450nm, 540nm and 610nm. It is intuitive that in order to achieve similar numbers the sensors must be strongly peaked at these measurement wavelengths.

There are, in fact, many other ways in which sensors satisfying 10 might be discovered. Finlayson et al[4] presented 3 methods for spectral sharpening. Sensor based sharpening is a method for finding the linear combination of sensors which is maximally sensitive within a given wavelength band. In the limiting case if an infinitesimal wavelength band is chosen then the transform is again defined by (13). The best transform might also be discovered

directly from Equation (12). What sensor transform best supports a diagonal matrix model of illuminant change? Data-based sharpening answers this question through optimisation but again returns sensors similar to those shown in Figure 3a. Perfect sharpening sets forth algebraic constraints for which (12) holds and again sensors similar to 3a are discovered. These constraints were later generalised[3] and more recently other constraints have been investigated[2, 11]. In all cases similar transforms and sensors are discovered and Equation (12) was found to be an excellent model of illumination change.

The sharp transform which is justifiable through mathematical/engineering argument finds support in the psychophysical and colour science literature. In colour science colour adaptation transforms attempt to model colour response across illumination change. Sharp transforms of the form (12) (and being based on sensors similar to those shown in Figure 3a) have been found to account for corresponding colour paper[14, 7] and importantly to improve over (in terms of modelling error) antecedent methods. Sharp sensors are also found in a variety of psychophysical experiments including the determination of test- and field-spectral sensitivity[20, 8, 19, 9, 12] and investigations into colour discrimination[16].

4. Colour Matching

Since we ourselves are trichromatic, it follows that we can reproduce the sensation of colour with three primary stimuli. A single primary induces a single l, m and s response. A second primary yields a second response vector and a third response vector is induced by a third primary. Since Equation (1) is linear, it follows that linear combinations of the primaries result in linear combinations of the corresponding cone response vectors. Given monochromatic primaries of unit power anchored at wavelengths i, j and k, the response of the eye is equal to:

$$\mathcal{T} = \begin{bmatrix} L(\lambda_i) & L(\lambda_j) & L(\lambda_k) \\ M(\lambda_i) & M(\lambda_j) & M(\lambda_k) \\ S(\lambda_i) & S(\lambda_j) & S(\lambda_k) \end{bmatrix} \quad (14)$$

where each column contains the response to a single primary. The response to a monochromatic test light at wavelength z is equal to $\underline{l}_z = [L(\lambda_z) \;\; M(\lambda_z) \;\; S(\lambda_z)]^t$. In color matching we seek the linear combination $\underline{x} = [x_1 \;\; x_2 \;\; x_3]^t$ of the columns of $\mathcal{T}$ (and hence of the primary lights) that induces the same response $\underline{l}_z$. Simple linear algebra implies that:

$$\mathcal{T}\underline{x} = \underline{l}_z \quad (15a)$$

$$\underline{x} = \mathcal{T}^{-1}\underline{l}_z \quad (15b)$$

That is, the relative powers of the primaries needed to achieve a match (defined as a component of the 3-vector $\underline{x}$) is a linear transform $\mathcal{T}^{-1}$ of the corresponding cone response. Taking the test light across the visible spectrum we get one coefficient vector per wavelength. The values of the three coefficients are plotted as a function of wavelength in Figure 3b (assuming primaries at 450nm, 540nm and 610nm). Again because (1) is linear it is straightforward to show that the mixture of primaries needed to match an arbitrary stimulus can be found by integrating the stimulus with respect to the matching curves shown in Figure 3a.

At this point the reader might wonder if a mistake has been made. The matching curves shown in 3b are **identical** to the sharp sensors shown in Figure 3. No mistake has been made. Mathematical argument which supports a simple diagonal model of illuminant change results in sensors that are also matching curves. The practical import of this is that if a camera is equipped with sharp sensors, then illumination is readily discounted using a diagonal matrix resulting in values that can *directly* drive a display without further matrixing.

The astute reader may be concerned that we have performed a sleight of hand. We have derived matching curves assuming primary wavelengths anchored at those wavelengths found to be optimal for adaptation calculation. But, are these wavelengths themselves the most appropriate choice of primaries for a colour display? To answer this question we might consider two criterion: gamut size and energy efficiency. In carrying out colour matching we can only use positive coefficients (the components of $\underline{x}$ must be greater than zero). The import of this is that a 3 primary CRT can only match a subset of all the colours we might see. The size of the gamut of colours we can reproduce is therefore a useful criterion to judge the usefulness of a CRT. Similarly, energy efficiency is also important. A monitor which could reproduce many colours but consumed a large amount of power would not be practical. Ideally, we would like to have a monitor that consumed as little power as possible: used as few Watts as possible to match colours.

It has been shown elsewhere[1] that, assuming monochromatic primaries, that the largest gamut is induced for primaries anchored close to 450nm, 540nm and 610nm. Moreover, these same primaries are optimal in terms of energy efficiency.

5. Conclusion

Spectral sharpening is a method for finding linear combinations of cone functions optimally sensitive to a particular band of the visible spectrum. The sharpest sensors (those which are relatively the most sensitive to light at 450nm, 540nm and 610nm) are those that best account for metameric matching data, support optimal von Kries type

colour adaptation and are also matching curves for a monitor with primaries at 450nm, 540nm and 610nm. Such a monitor is optimally energy efficient and has the largest possible colour gamut.

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