A Novel Method for Computing Optimum Weights for Calculating CIE Tristimulus Values

C. J. Li, M. R. Luo and B. Rigg
Colour & Imaging Institute, University of Derby
Derby, United Kingdom

Abstract

A study by Fairman showed that the optimum weights calculated by Venable's method performs better in most cases in comparison with those calculated using the Stern's correction method. However, the Venable method is an iterative method. Although these weights are effective in reducing computation errors, the method is difficult to implement. In this study, a simple analytical method is described for computing optimum weights. Its performance was tested together with the other methods. The results showed that the optimum weights from the new method performs equally well or better compared with the Venable's optimum weights.

Introduction

Tristimulus values are the basis of colorimetry. They are transformed to colorimetric coordinates for industrial applications such as colour specification, colour quality control and recipe formulation. Accurate calculation of tristimulus values is highly desired by industry for open environment applications such as colour management between different computer platforms across different input and output imaging devices.

CIE (International Committee on Illumination)[1,2] originally defines the tristimulus values in terms of the integrals:

$$\begin{aligned} X &= k\int_a^b S(\lambda)\bar{x}(\lambda)R(\lambda)d\lambda \\ Y &= k\int_a^b S(\lambda)\bar{y}(\lambda)R(\lambda)d\lambda \\ Z &= k\int_a^b S(\lambda)\bar{z}(\lambda)R(\lambda)d\lambda \end{aligned} \qquad (1)$$

where $S(\lambda)$ is the relative spectral power distribution of an illuminant, $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ are the colour matching functions of CIE 1931 or 1964 standard observer, and $R(\lambda)$ is the reflectance function of a colour. k is a normalizing factor. (a,b) is the visible range of the spectral wavelength, i.e. a = 360 nm and b = 830 nm. The CIE[1] defines only the colour matching functions and the relative spectral power distributions of some illuminants at 1-nm intervals. Thus, although the CIE defines the above tristimulus values as integrals of the continuous functions: $S(\lambda)$, $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ and $R(\lambda)$, the analytical expressions for them are not available. Therefore, to calculate the tristimulus values becomes a problem. Between 1981 and 1983, Working Group VIII [3] of U.S was formed to tackle this problem. Finally, they recommended that the definition of the CIE tristimulus values be based on numerical rather than analytical integration and the integration may be carried out by numerical summation at wavelength intervals $\Delta\lambda$ = 1 nm, which were adopted by the CIE.[2] Thus, the alternative practice for calculating the tristimulus values by CIE[2] is the following:

$$\begin{aligned} X &= k\sum_{i=0}^{n} S(\lambda_i)\bar{x}(\lambda_i)R(\lambda_i)\Delta\lambda \\ Y &= k\sum_{i=0}^{n} S(\lambda_i)\bar{y}(\lambda_i)R(\lambda_i)\Delta\lambda \\ Z &= k\sum_{i=0}^{n} S(\lambda_i)\bar{z}(\lambda_i)R(\lambda_i)\Delta\lambda \end{aligned} \qquad (2)$$

with

$$\Delta\lambda = \frac{b-a}{n},\ \ \lambda_i = a + i\Delta\lambda,\ \ i = 0,1,\cdots,n \ \text{ and } \ \Delta\lambda = 1.$$

Reflectance Measurements

In eq. (2), the reflectance function values at points λ_i, i = 0,1,…,n, of an object colour at 1-nm intervals must be known. However, they are not available in practice, since they generally are measured in larger intervals such as $\Delta\lambda$ = 5, 10, or 20 nm with a spectrophotometer. Hence there is a need to transform the measured data to the desired reflectance values at 1-nm intervals. Note that a spectrophotometer is an instrument that can be regarded as making a sequence of observations, each of which involves a very limited spectral range. If we let R'_i be the value measured, then it satisfies

$$R'_i = \int_a^b P_i(\lambda)R(\lambda)d\lambda\,, \qquad (3)$$

where $P_i(\lambda)$ is the instrumental function of a spectrophotometer at a setting number i corresponding to the wavelength at λ_i and the integration (eq. (3)) is carried

out over the entire visible wavelength region. In practice, $P_i(\lambda)$ is scaled to make $R'_i = 1$ for a perfect diffuser, i.e.,

$$\int_a^b P_i(\lambda)d\lambda = 1\,,\ i = 0,1,\cdots,n\,. \tag{4}$$

For the purpose of this study, it is assumed that $P_i(\lambda)$ is symmetrical and triangular in shape, and its half-peak width is equal to the measurement interval as shown in Fig. 1. It represents the most common situation encountered in industrial practice.[4-7] Thus,

$$P_i(\lambda) = \begin{cases} (\lambda - \lambda_{i-1})/(\Delta\lambda)^2, & \text{when } \lambda_{i-1} \le \lambda \le \lambda_i \\ (\lambda_{i+1} - \lambda)/(\Delta\lambda)^2, & \text{when } \lambda_i \le \lambda \le \lambda_{i+1} \\ 0, & \text{when } \lambda < \lambda_{i-1} \text{ or } \lambda > \lambda_{i+1} \end{cases} \tag{5}$$

Note that when the reflectance function $R(\lambda)$ is a linear function of wavelength within the range of bandpass, then the measured values R'_is are the same as the true reflectance values $R(\lambda_i)$. However, when $R(\lambda)$ is not a linear function of wavelength within the range of bandpass, then there are differences between the measured values R'_is and the true values $R(\lambda_i)$. This difference is called the bandpass effect. Sterns and Sterns[8] and Venable[6] proposed methods for correcting the bandpass effect. It is clear that if eq. (2) is used to calculate the tristimulus values, it is not possible to obtain the desired reflectance values at 1-nm intervals by directly interpolating the measured data R'_is. There is a need to correct the measured data R'_is using the Sterns and Sterns (SS) correction[8] before the interpolation. The Stearns and Stearns bandpass dependence correction formula is given by

$$R_i = -\alpha R'_{i-1} + (1+2\alpha)R'_i - \alpha R'_{i+1}\,, \tag{6}$$

where α = 0.083. When correcting the bandpass dependence for the first or the last measured bandpass, eq. (7) should be used:

$$R_i = (1+\alpha)R'_i - \alpha R'_{i\pm1} \tag{7}$$

Weighting Tables

From eq. (2) it can be seen that only the reflectance values change with the object colour, all other quantities are unchanged. It is possible therefore that the products of these unchanged quantities can be pre-calculated for calculating tristimulus values of the object colours for a particular illuminant-observer combination. These products depend on the illuminant-observer and are called weights and a set of them under the same illuminant-observer is called the weighting table. Thus, using the CIE standard illuminants and observers in 1-nm intervals some 1-nm weighting tables can be established. Let $W^{(1)}_{X,i}$, $W^{(1)}_{Y,i}$, $W^{(1)}_{Z,i}$ be the weights, then

$$\begin{aligned} W^{(1)}_{X,i} &= kS(\lambda_i)\bar{x}(\lambda_i) \\ W^{(1)}_{Y,i} &= kS(\lambda_i)\bar{y}(\lambda_i)\,,\quad i = 0,1,\cdots,n \\ W^{(1)}_{Z,i} &= kS(\lambda_i)\bar{z}(\lambda_i) \end{aligned} \tag{8}$$

Here the superscript (1) means weights in 1-nm intervals. Thus, if the reflectance values $R_i = R(\lambda_i)$ of the object colour are known, then the tristimulus values are given by:

$$\begin{aligned} X &= \sum_{i=0}^{n} W^{(1)}_{X,i} R_i \\ Y &= \sum_{i=0}^{n} W^{(1)}_{Y,i} R_i \\ Z &= \sum_{i=0}^{n} W^{(1)}_{Z,i} R_i \end{aligned} \tag{9}$$

However, current industrial practice requires weighting tables at 10-nm and 20-nm intervals. After an object colour is measured by a spectrophotometer, the measurement reflectance data (eq. (3)) or its corrected data using eqs. (6) and (7)), and the corresponding weighting table are used to do the similar summation in eq. (9) to obtain the tristimulus values. Hence there is a need to have weighting tables with larger intervals derived from the corresponding 1-nm interval weighting tables. The first set of (larger intervals) weighting tables were calculated by Foster *et al*[10] and Stearns.[11] In 1985, the American Society for Testing and Materials (ASTM) [12] recommended a set of weighting tables as the ASTM E308-85 standard. These tables should be used with reflectance values previously corrected for bandpass dependence using the Stearns and Stearns' correction. These tables were obtained by first interpolating the reflectance values in 10-nm or 20-nm to obtain the reflectance values in 1-nm intervals, then using the summation (9), and finally reorganizing the summation in terms of the reflectance values in 10-nm or 20-nm intervals. In 1989, Venable[6] derived an iterative computation procedure by which a correction for spectral bandpass effect could be built into weights directly. This would allow calculation, once for all, of both the weights and the spectral bandpass correction, and users would integrate spectral data as yet uncorrected for spectral bandpass effect with the resulting set of weights and obtain tristimulus values that were then corrected for spectral bandpass effect. In 1995, Fairman[4] pointed that the Sterns and Sterns correction can also be applied to the weighting tables and the resulting tables can be used with the measured reflectance data (uncorrected data) to give the tristimulus values corrected by the Sterns and Sterns method. Fairman named this kind of tables as Sterns correction tables. Furthermore, he found that the Venable's weighting tables out-performs the Sterns correction tables at 10-nm interval for continuous illuminants, but the opposite for fluorescent illuminants. He also found that the Venable weighting tables outperform the Sterns correction tables at 20-nm interval for all illuminants tested. Therefore, an mixed strategy was proposed, i.e., the Venable's weighting tables at 20-nm interval are used for all illuminants, while for 10-nm interval the Venable's weighting tables are used for continuous illuminants and the Sterns correction tables are used for the fluorescent illuminants. In 1995, ASTM adopted the above strategy and published another set of weighting tables to be used for reflectances that have not been corrected for bandpass dependence. Both sets of weighting tables were included in ASTM E308-95 version,[7] called weighting tables of Tables 5 and 6 for

applying to reflectances with and without bandpass dependence correction respectively.

The ASTM only provided weighting tables for some standard illuminants. However, in practice the illuminant used may be different from the standard illuminant. Therefore, users have to prepare their own tables. However, there is no standard procedure to produce Venable's weights since it is an iterative procedure. Hunt[5] pointed that "it is clear that the optimum weights (Venable's weights) will be different for each combination of observers, wavelength interval, bandwith function (instrument function) and illuminant, and they are not easy to calculate; but they can be very effective in reducing errors". Thus, there is a need to provide a simpler method to produce the Venable's weighting tables, which is the main aim of this paper and will be discussed next.

The Proposed Algorithm for Calculating Optimum Weighting Tables

In order to save space, we will outline the algorithm only and omit the details for deriving the algorithm.

Let A be the tridiagonal matrix of order (n + 1). Its diagonal elements are all equal to 4, and all elements of its upper and lower sub-diagonals are equal to 1. And let $w(\Delta\lambda,V)^1 = (W(\Delta\lambda,V,0), W(\Delta\lambda,V,1),\ldots,W(\Delta\lambda,V,n))$ with $V = X,Y$, and Z. Note that $W(\Delta\lambda,V,i)$ are the weights wanted at $\Delta\lambda$ nm interval. For example, $\Delta\lambda$ can be 10 or 20. Once $\Delta\lambda$ is determined, the integer n can be determined by using $n = (b - a)/\Delta\lambda$.

Now the algorithm for computing weights $W(\Delta\lambda,V,i)$, $i = 0,1,\ldots,n$ with $V = X, Y$, and Z respectively can be described below.

Algorithm: Computing the Optimum Weights $W(\Delta\lambda,X,i)$, $W(\Delta\lambda,Y,i)$, $W(\Delta\lambda,Z,i)$, $i = 0,1,\ldots,n$

Step 1: Input

Input the CIE colour matching functions $\bar{x}(j),\bar{y}(j),\bar{z}(j)$ and the spectral power distribution $S(j)$ of the illuminant used with $j = a, a + 1, a + 2, \ldots, b$, and input $\Delta\lambda$, the length of the interval for the wanted weights and determined the integer n and with $\lambda_j = a + j\Delta\lambda$ with $j = 0,1,\ldots,n$.

Step 2: Compute Weights $W(\Delta\lambda,X,i)$, $i = 0,1,\ldots,n$

Compute $b_i = 6\Delta\lambda k\int_{\lambda_{i-1}}^{\lambda_{i+1}} S(\lambda)\bar{x}(\lambda)P_i(\lambda)d\lambda$, $i = 0, 1, \ldots, n$, form a column vector b with b_is computed, and solve $Aw(\Delta\lambda,X) = b$ for weights $W(\Delta\lambda,X,i)$, $i = 0,1,\ldots,n$.

Step 3: Compute Weights $W(\Delta\lambda,Y,i)$, $i = 0,1,\ldots,n$

Compute, $b_i = 6\Delta\lambda k\int_{\lambda_{i-1}}^{\lambda_{i+1}} S(\lambda)\bar{y}(\lambda)P_i(\lambda)d\lambda$, $i = 0, 1, \ldots, n$, form a column vector b with the b_is computed, and solve $Aw(\Delta\lambda,Y) = b$ for weights $W(\Delta\lambda,Y,i)$, $i = 0,1,\ldots,n$.

Step 4: Compute Weights $W(\Delta\lambda,Z,i)$, $i = 0,1,\ldots,n$

Compute, $b_i = 6\Delta\lambda k\int_{\lambda_{i-1}}^{\lambda_{i+1}} S(\lambda)\bar{z}(\lambda)P_i(\lambda)d\lambda$, $i = 0, 1, \ldots, n$, form a column vector b with the b_is computed, and solve $Aw(\Delta\lambda,Z) = b$ for weights $W(\Delta\lambda,Z,i)$, $i = 0,1,\ldots,n$.

Step 5: Output Weights $W(\Delta\lambda,X,i)$, $W(\Delta\lambda,Y,i)$, $W(\Delta\lambda,Z,i)$ $i = 0,1,\ldots,n$

Note that the function $P_i(\lambda)$ in the above Algorithm is the instrumental function defined by eq. (5). Since the colour matching functions and the spectral power distribution of the illuminant are defined at 1nm intervals, therefore numerical integration technique should be used for integrations in the above algorithm. Note also that Venable proposed an iterative method for obtaining the optimum weights. It is clear that the above algorithm is much simpler to implement.

Conclusions

A simple analytical method for computing the Venable's optimum weights is derived. It was tested together with the other methods. The results in Tables 1 to 6 consistently in terms of CIELAB ΔE values showed that the optimum weights computed by the new algorithm performs equally well or better than the weighting tables of ASTM Table 6, which was calculated using either the Venable's optimum weights or ASTM Table 5 corrected using Sterns and Sterns method as proposed by Fairman.[4] In many industrial applications, the real illuminants are different from the standard CIE illuminants. Hence, there is no standard weighting table at larger intervals, say at 10 or 20 mn intervals. Therefore, the proposed method can produce weighting tables for practical industrial applications.

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Biography

Dr. Changjun Li is a Research Fellow at the Colour & Imaging Institute, University of Derby, UK. He received his B.Sc. in computational mathematics from Peking University, China in 1979, M.Sc. in numerical analysis from the Department of Technical Sciences, Chinese Academy of Science in 1982 and Ph.D. at the Department of Computer Studies of Loughborough University, UK in 1989. He was Professor of computational mathematics in the Northeastern University of China before joining the Colour & Imaging Institute. He received the Scientific Progress First Prize awarded by the Educational Committee of Liaoning Province of China in 1998.

Dr. M. Ronnier Luo is the Director of the Colour & Imaging Institute and Professor of Colour Science at University of Derby, UK. He received his B.Sc. in Fiber Technology from the National Taiwan Institute of Technology in 1981 and his Ph.D. in Colour Physics from the University of Bradford in 1986. He has published over 120 papers in the field of colour science. He is the Chairman of the Colour Measurement Committee (CMC) of the Society of Dyers and Colourists (SDC), and the CIE TC 1-52 on Chromatic Adaptation Transforms. He was the recipient of the 1994 Bartleson award for his work in colour science.

Table 1. Performances of the weighting tables from ASTM Tables 5 and 6, and the optimum at 10 nm and 20 nm intervals for illuminant: D65 (CIE ΔE^*_{ab})

Weighting Tables	10nm interval			20nm interval		
	Min	Max	Median	Min	Max	Median
Table 5	0.00015	0.02806	0.00731	0.00422	0.37647	0.07393
Table 6	0.00011	0.00862	0.00159	0.00096	0.05069	0.01595
Optimum	0.00008	0.00557	0.00107	0.00044	0.04511	0.01466

Table 2. Performances of the weighting tables from ASTM Tables 5 and 6, and the optimum at 10nm and 20nm intervals for illuminant: D50 (CIE ΔE^*_{ab})

Weighting Tables	10nm interval			20nm interval		
	Min	Max	Median	Min	Max	Median
Table 5	0.00061	0.02507	0.00732	0.00172	0.36079	0.07475
Table 6	0.00029	0.01873	0.00400	0.00050	0.05796	0.01225
Optimum	0.00002	0.00477	0.00123	0.00034	0.05263	0.01266

Table 3. Performances of the weighting tables from ASTM Tables 5 and 6, and the optimum at 10nm and 20nm intervals for illuminant: A (CIE ΔE^*_{ab})

Weighting Tables	10nm interval			20nm interval		
	Min	Max	Median	Min	Max	Median
Table 5	0.00026	0.02158	0.00515	0.00202	0.32652	0.06329
Table 6	0.00025	0.01010	0.00301	0.00059	0.04748	0.01257
Optimum	0.00005	0.00414	0.00114	0.00028	0.04514	0.01340

Table 4. Performances of the weighting tables from ASTM Tables 5 and 6, and the optimum at 10nm and 20nm intervals for illuminant: F2 (CIE ΔE^*_{ab})

Weighting Tables	10nm interval			20nm interval		
	Min	Max	Median	Min	Max	Median
Table 5	0.00010	0.05144	0.00875	0.00203	0.49224	0.11500
Table 6	0.00055	0.05904	0.00875	0.00204	0.64219	0.05561
Optimum	0.00007	0.02254	0.00232	0.00086	0.32076	0.02966

Table 5. Performances of the weighting tables from ASTM Tables 5 and 6, and the optimum at 10nm and 20nm intervals for illuminant: F7 (CIE ΔE^*_{ab})

Weighting Tables	10nm interval			20nm interval		
	Min	Max	Median	Min	Max	Median
Table 5	0.00053	0.05305	0.00876	0.00389	0.57771	0.09616
Table 6	0.00006	0.05660	0.00764	0.00218	0.58001	0.05161
Optimum	0.00006	0.02182	0.00226	0.00051	0.31818	0.03397

Table 6. Performances of the weighting tables from ASTM Tables 5 and 6, and the optimum at 10nm and 20nm intervals for illuminant: F11 (CIE ΔE^*_{ab})

Weighting Tables	10nm interval			20nm interval		
	Min	Max	Median	Min	Max	Median
Table 5	0.00072	0.29392	0.03189	0.01260	2.58160	0.28554
Table 6	0.00306	0.26887	0.03624	0.00561	1.31776	0.21685
Optimum	0.00007	0.04514	0.00570	0.00406	1.39941	0.18088

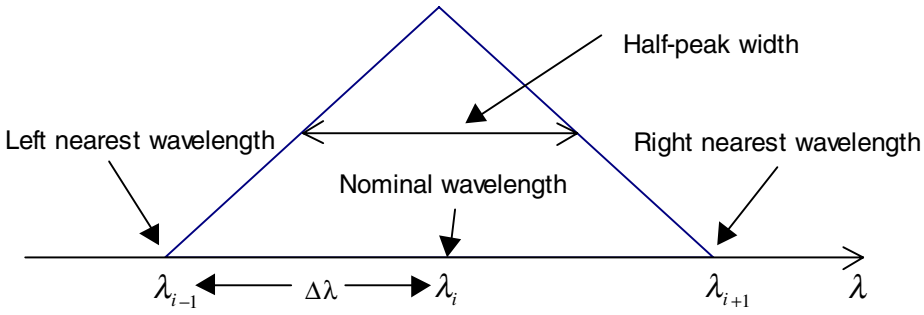

Figure 1.Instrumental Function $P_i(\lambda)$