Chromaticity Difference from Surfaces Defined from MacAdam Ellipses

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Abstract

We have developed a model to calculate chromaticity difference from a surface defined from the MacAdam ellipses in CIE 1931 (x,y)-chromaticity diagram. The idea is to tranform the MacAdam ellipses to circles on a surface. The height of the surface is obtained by projecting a circle of constant radius such that the projection is equal to the ellipse on the the (x,y)-plane. The chromaticity difference is calculated from the surface by a method based on the Weighted Distance Transform On Curved Space (WDTOCS).

Introduction

The MacAdam chromaticity difference ellipses are widely used in the chromaticity difference calculations. The area inside each ellipse appears as an equal color so that the chromaticity differences cannot be perceived.[1] The ellipses have various sizes in different parts of the horseshoe shaped spectral locus, thus the calculated chromaticity differences in each part of the locus are perceived unequally.[2] For example, in the bottom left corner in the blue area small Euclidean distance yields to a large chromaticity difference and in upper part of diagram in the green area the same chromaticity difference results in a much larger Euclidean distance. MacAdam has made also a model for chromaticity difference calculations.[3]

Several color difference models have been developed, they try to compensate the nonuniform size of the ellipses in the (x,y)-plane. The CIELAB and CIELUV models were among the first one.[4] Then CIE94 was developed with a set of variables for the parametric correction of the error from the CIELAB ΔE formula. The CMC model for textile industry is dividing the ab-plane into microfacets thus compensating the planar color difference errors.[4]

Our purpose was to develop a model, which gives an equal perceived difference in every part of the CIE-1931 (x,y)-chromaticity diagram for equal chromaticity differences.

Defining the Model

Defining the Surfaces

The chromaticity differences are calculated from the surface which is defined from the MacAdam ellipses. The surface is based on the parameters of the 25 ellipses, see Figure 1.

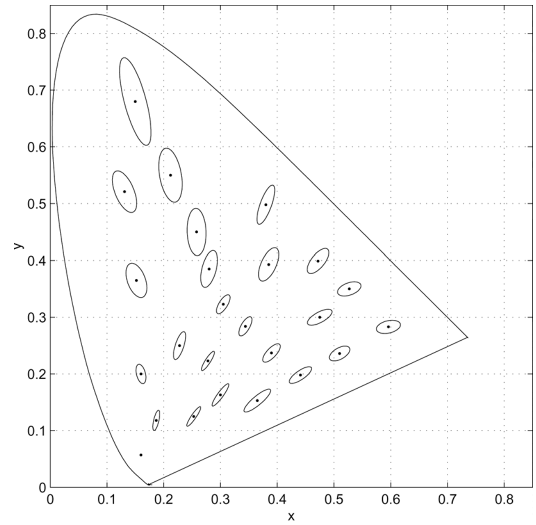

Figure 1. The MacAdam ellipses. The axes of plotted ellipses are 10 times their actual lengths.[5]

The main idea is that the ellipses are projections of circles on the surface. Every circle is projected from the center of projection above each circle. The centers of projection are called illumination points and they all are at same height H from the (x,y)-plane. The height h of surface is obtained for each ellipse depending on the of each ellipse. For a large ellipse the projected circle closer to the illumination point and thus the height h gets a small value. For a small ellipse the height h gets a larger value. Two different surfaces are defined, the first one based on the major semiaxes a and the second one on minor semiaxes b of the ellipses, see Figures 2 and 3. In the previous case, the projected circle has radius $r = r_a$ and in the latter case the projected circle has radius $r = r_b$.

The surface S_b defined from the minor semiaxis b lies higher than the surface S_a defined from major semiaxis a, because the height h of the surface is measured from the illumination point. The surface S used in the calculation of chromaticity differences is a mixture of these two surfaces defined as

$$S = pS_b + (1 - p)S_a \tag{1}$$

where the parameter $p = p(\theta_1, \theta_2)$, $0 \leq p \leq 1$. The angle θ_1 depends on the orientation of the two chromaticities and the angle θ_2 is the average value of the angles of the two ellipses closest to the two chromaticities, see Figure 4.

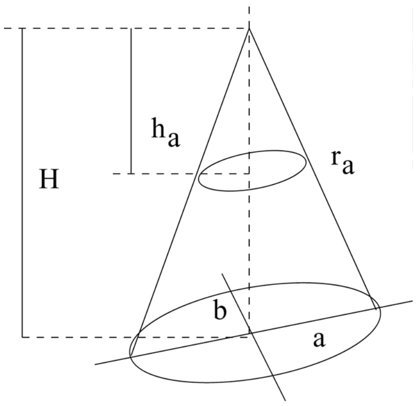

Figure 2. Projection of the circle, major semiaxis.

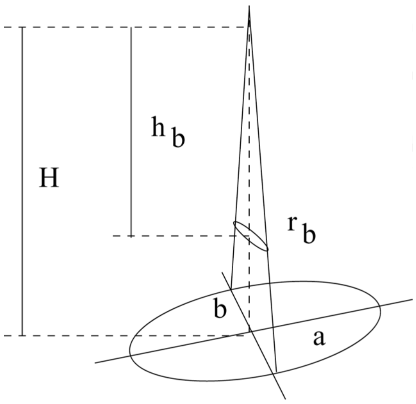

Figure 3. Projection of the circle, minor semiaxis.

The angles θ_1 and θ_2 depend on the two chromaticities, whose difference will be calculated. The coordinates of these two chromaticities are (x_0, y_0) and (x_1, y_1), see Figure 4. The average value $\Delta\theta$ is calculated from the two closest ellipses for both of the chromaticities. Then p is angle between θ_1 and θ_2 calculated as

$$p = \frac{\Delta\theta}{\pi/2} \; if \;\; \Delta\theta \leq \pi/2 \tag{2}$$

or otherwise as

$$p = \frac{|\Delta\theta - \pi|}{\pi/2} \tag{3}$$

where

$$\Delta\theta = |\theta_1 - \theta_2| \tag{4}$$

If $\theta_1 = \theta_2$ then $p = 0$ and the orientation of the chromaticities is parallel to the orientation of the ellipses major semiaxis in that area. Now the surface S consists only the surface S_a. If the line between the two chromaticities is perpendicular to the major semiaxis then $p = 1$ and surface S consists only of the surface S_b. Normally, surface S is a mixture of the both surfaces S_a and S_b, Figure 5.

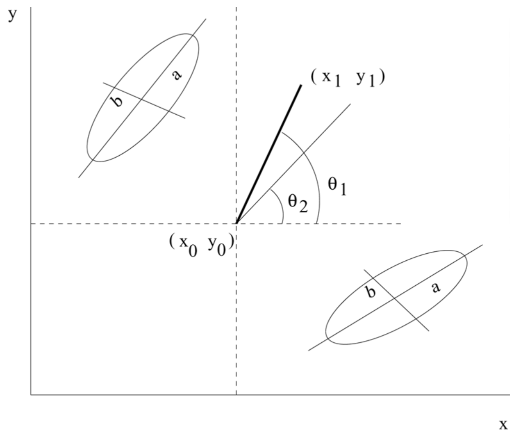

Figure 4. Angles θ_1 and θ_2 in the calculation of the parameter

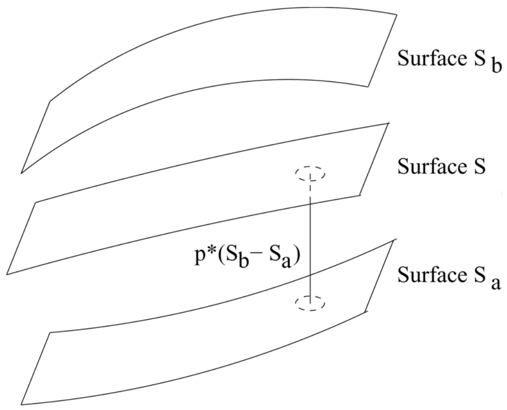

Figure 5. Defining surface S from the surfaces S_a and S_b.

In Figure 6, we show the surfaces S_a and S_b. The surface S_b is elevated by 0:03 units for better visualization the two surfaces. These surfaces are not dependent on chromaticities selected for the difference calculation.

In Figure 7 we illustrate the surface S calculated from two chromaticities $(x_0, y_0) = (0, 304, 0, 433)$ and $(x_1, y_1) = (0, 314; 0, 453)$. The surface S depends on the two chromaticities selected, and it is accurate only in the vicinity these chromaticities.

The MacAdam ellipses cover the center of the horseshoe shaped diagram, but the edge of the horseshoe to be defined in another way. We extrapolated the edge of the horseshoe on the basis of the contour diagrams the covered area in the CIE 1931 (x,y)-chromaticity diagram. We examined the CIE-diagram to decide where surface rises near the edge and where it falls. We assumed that there were not any irregularities near the edge, but the slopes were in harmony with the covered areas. Another method is to extrapolate the edges through the Just-Noticeable-Differences (JNDs). JNDs are defined in the spectral locus and JNDs are three times larger than the corresponding standard deviation from the MacAdam ellipses.[1,5]

All the ellipse parameters are used in creation of the surface S. The major and minor semiaxis define the two surfaces and the surface used in the chromaticity difference calculation is a mixture of these two surfaces depending on the orientation of the selected pair of chromaticities.

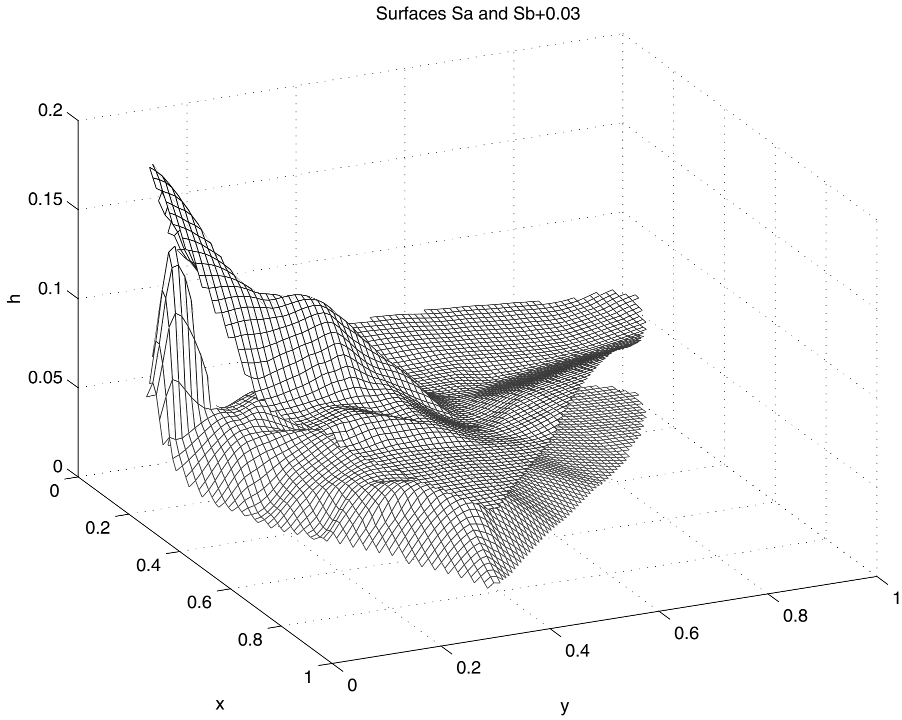

Figure 6. Surfaces S_a and S_b, which was elevated for visualizing, see text.

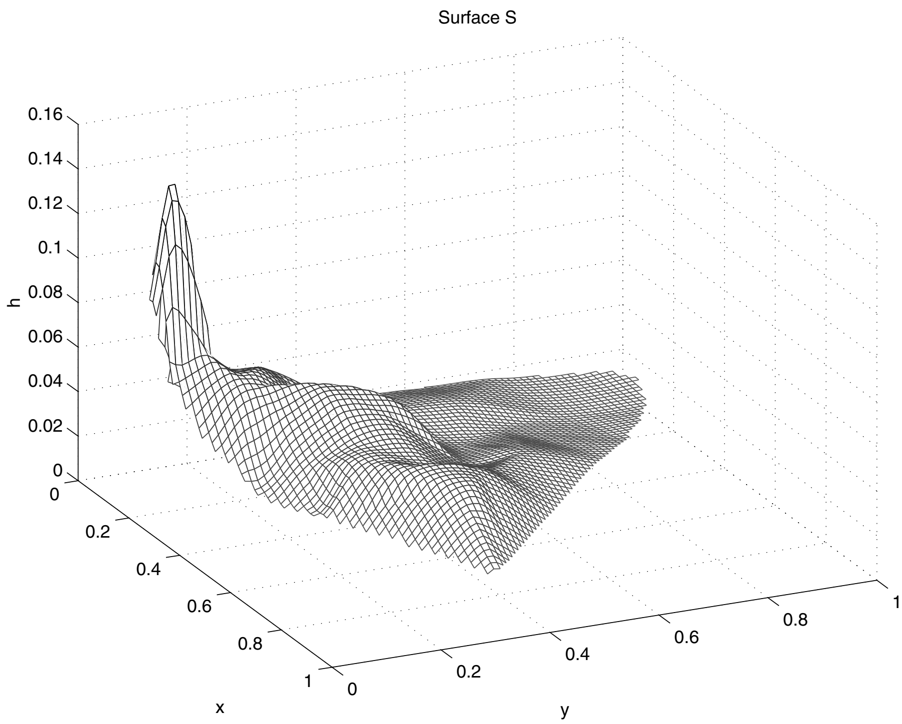

Figure 7. Surface S according to the two chromaticities (x_0, y_0) and (x_1, y_1), see text.

Calculating the Distance on a Curved Surface

The chromaticity difference is calculated as

$$\Delta E = \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2 + \alpha f(S)} \tag{5}$$

where $f(S)$ contains the contribution of the surface S. We applied theWeighted Distance Transform On Curved Space,[6] where ΔE was replaced by partial Euclidean distances. In WDTOCS, every subdistance between neighboring pixels was Euclidean, but the whole distance between the two chromaticity points was not.

The Weighted Distance Transform on Curved Space (WDTOCS) between two points is defined as the minimum of all possible paths linking those points. The WDTOCS algorithm requires only two passes over the image with a chosen kernel. In order to implement the WDTOCS algorithm, two surface models are needed: the original gray-level image, and the second image, which determines the region or regions in which the transform is calculated. The transform is performed on this image. Now, the horseshoe shaped area constitutes the region for calculation and we have to select one point from that area as the starting point. This point is one chromaticity point and after the calculation, the distances to all other chromaticities are obtained.

a	b	c
d	e	f
g	h	k

Figure 8. The kernel for the WDTOCS calculation.

The algorithm, which applies the WDTOCS, proceeds as follows. Let $\mathcal{G}(x)$ denote the original gray-level image and let $\mathcal{A}(x)$ denote the binary image which determines the region(s) in which the transform is calculated. $\mathcal{F}^*(x)$ means an already calculated point. $\mathcal{F}^*(e)$ denotes the new distance value of the point e in the image $\mathcal{F}$. Let $N_4(e)$ denote the 4 horizontal and vertical neighbors of a pixel e similarly as in the city block kernel. $\mathcal{G}(e)$ denotes the gray value of the center point in the 3 x 3 kernel and $\mathcal{G}(x_i)$ denotes the gray values of the pixels $x_i \in N_4(e)$. The kernel is depicted in Figure 8.

1st Iteration

The first iteration round proceeds "direct video order" (from top to bottom, and right) calculating the new point $\mathcal{F}^*(e)$. The points with asterix * hold already once calculated distance values while the point $\mathcal{A}(e)$ has the initial value, which maximal representative integer number. The iteration proceeds as follows:

$$\begin{aligned}\mathcal{F}^*(e) = \min[&\mathcal{A}(e), \min(da + \mathcal{F}^*(a), db + \mathcal{F}^*(b),\\ &dc + \mathcal{F}^*(c), dd + \mathcal{F}^*(d))]\end{aligned} \tag{6}$$

where

$$da = \alpha\sqrt{(\mathcal{G}(e) - \mathcal{G}(a))^2 + \beta}$$
$$db = \alpha\sqrt{(\mathcal{G}(e) - \mathcal{G}(b))^2 + \delta}$$
$$dc = \alpha\sqrt{(\mathcal{G}(e) - \mathcal{G}(c))^2 + \beta}$$
$$dd = \alpha\sqrt{(\mathcal{G}(e) - \mathcal{G}(d))^2 + \delta}$$

The parameter values β = 2 and δ = 1 are corresponding values in the WDTOCS definition.

2nd Iteration

The second iteration round proceeds in the "inverse video order" (from bottom to up, right to left) calculating the new point $\mathcal{F}^*(e)$. The points marked with asterix * hold already once calculated distance values while the point $\mathcal{A}(e)$ has a value obtained when applying Equation 6. The second iteration as follows.

$$\begin{aligned}\mathcal{F}^*(e) = \min[&\mathcal{F}(e), \min(df + \mathcal{F}^*(f), dg + \mathcal{F}^*(g),\\ &dh + \mathcal{F}^*(h), dk + \mathcal{F}^*(k))]\end{aligned} \tag{7}$$

where

$$df = \alpha\sqrt{(\mathcal{G}(e) - \mathcal{G}(f))^2 + \delta}$$
$$dg = \alpha\sqrt{(\mathcal{G}(e) - \mathcal{G}(g))^2 + \beta}$$
$$dh = \alpha\sqrt{(\mathcal{G}(e) - \mathcal{G}(h))^2 + \delta}$$
$$dk = \alpha\sqrt{(\mathcal{G}(e) - \mathcal{G}(k))^2 + \beta}$$

Again, β = 2 and δ = 1 corresponding to the WDTOCS definition.

Experimental Results

Defining the Independent Variables

There are three independent variables in our model. The height H of the illumination point, the radius r of the projected circle and the multiplier α in the distance measuring can be defined almost separately.

Now the radius r of the projected circle is the shortest of the major r_a and minor r_b semiaxis from MacAdam ellipses. The radius r cannot be longer than the shortest axis, since otherwise the projection is impossible or the circle would lie under the (x,y)-plane. Fixing the illumination point it is possible to define the surface in a desired height. The height H of the illumination point was derived from a computational procedure, where the best fit of the projections was found by varying the height H. Now, the illumination point was placed at the height H = 0:15. The multiplier α is now defined as α = 1. With this multiplier it is possible to change the weight of the surface in the calculation of the chromaticity difference.

Calculating Chromaticity Differences

The results show the influence of the surface S of the measured distance compared to the Euclidean distance on the (x,y)-plane. In Table 1 we show pairs of chromaticity coordinates from different parts of the horseshoe diagram with variable distance on (x,y)-plane.

Table 2 represents pairs of chromaticity coordinates with constant Euclidean distance on the (x,y)-plane with variable orientations and from different parts of the diagram.

In Table 3 we show the chromaticity coordinates with constant Euclidean distance from same locations as in Table 2, but the orientations are perpendicular to the orientations of the chromaticity pairs in Table 2.

In the Figures 9, 10, and 11 we illustrate the chromaticities, whose differences were calculated in the experiments. A line in (x,y)-plane is connecting each chromaticity pair.

The chromaticities plotted in Figure 9 are reported in Table 1, the chromaticities plotted in Figure 10 in Table 2, and the chromaticities plotted in Figure 11 are in Table 3. All the experiements were run in Matlab-environment.

The surface model acted as expected, now the chromaticity differences are closer to the perceived differences. This can be derived from the results in Tables 1, 2, and 3.

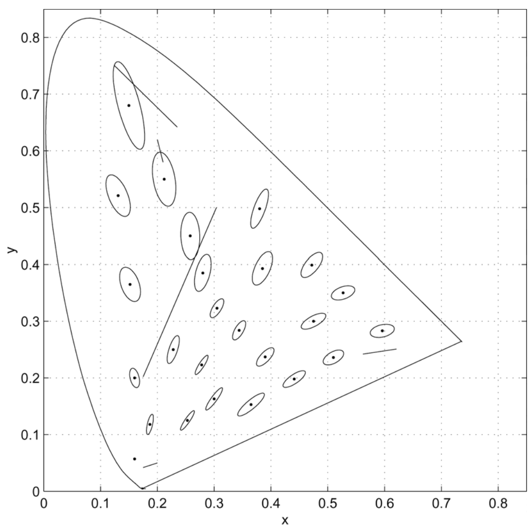

Figure 9. Chromaticities in the first experiment, the results are in Table 1.

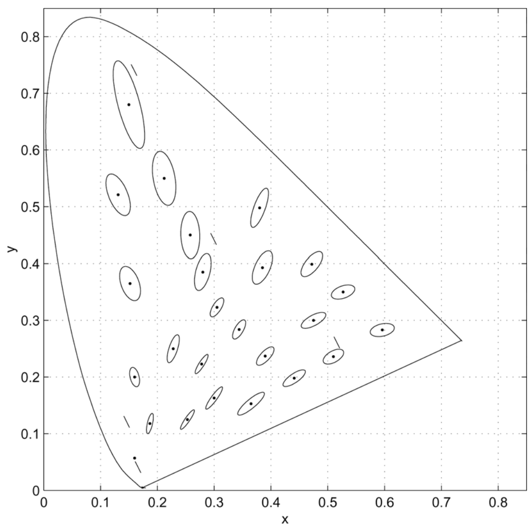

Figure 10. Chromaticities in the second experiment, the results are in Table 2.

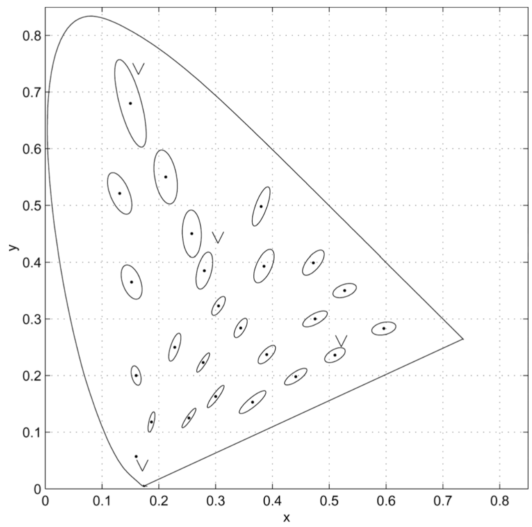

Figure 11. Chromaticities in the third experiment, the results are in Table 3.

Conclusions

We have developed a method to calculate chromaticity differences such that the calculated value matched with the perceived chromaticity difference. A surface based on the MacAdam ellipses was defined and the chromaticity difference was calculated from that surface by the Weighted Distance Transform on Curved Space. The surface varied according to the two chromaticities, whose difference under consideration.

The results show that the surface increases the chromaticity difference more in the bottom left corner, in blue area, than in the upper part, in the green area, of diagram, see Tables 1 and 2. This is natural, since the sizes of ellipses differ heavily in these two areas. From Table we can see that the orientation of the chromaticity pair affects the difference measured from the surface.

Our method can be applied to any set of ellipses. The projection principle is not fixed to the MacAdam ellipses, these were just used to illustrate the method. Thus, our method will produce better results also with other data available in the literature.

In the future, we will extend the method to color difference calculation thus including the effect of the illumination level.

Table 1. Points from different parts of diagram with variable distance on (x,y)-plane.

x0	y0	x1	y1	distance on (x,y)-plane	difference from surface S	relative gain
0.200	0.620	0.210	0.580	0.0412	0.0442	7.3
0.200	0.050	0.175	0.042	0.0262	0.0345	31.7
0.304	0.500	0.175	0.202	0.3247	0.3546	9.2
0.124	0.751	0.235	0.642	0.1556	0.1557	0.1
0.621	0.251	0.562	0.242	0.0597	0.0641	7.4

Table 2. Chromaticity coordinates with constant Euclidean distance on (x,y)-plane.

x0	y0	x1	y1	distance on (x,y)-plane	difference from surface S	relative gain
0.521	0.251	0.511	0.271	0.0224	0.0242	8.0
0.151	0.111	0.141	0.131	0.0224	0.0247	10.3
0.171	0.031	0.161	0.051	0.0224	0.0541	141.5
0.164	0.731	0.154	0.751	0.0224	0.0241	7.6
0.304	0.433	0.294	0.453	0.0224	0.0241	7.6

Table 3. Chromaticity coordinates with variable orientation in the (x,y)-plane.

x0	y0	x1	y1	distance on (x,y)-plane	difference from surface S	relative gain
0.164	0.731	0.174	0.751	0.0224	0.0242	8.1
0.164	0.731	0.154	0.751	0.0224	0.0241	7.6
0.171	0.031	0.181	0.051	0.0224	0.0308	37.5
0.171	0.031	0.161	0.051	0.0224	0.0541	141.5
0.521	0.251	0.531	0.271	0.0224	0.0241	7.6
0.521	0.251	0.511	0.271	0.0224	0.0242	8.1
0.304	0.433	0.314	0.453	0.0224	0.0242	8.1
0.304	0.433	0.294	0.453	0.0224	0.0241	7.6

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Biography

Arto Kaarna received his M.S. degree in 1980 in Mechanical Engineering. He received Licenciate of Science degree in 1990 and Doctor of Science degree in 2000 in computer science at Lappeenranta University of Technology, Finland (LUT). Currently he is working as a professor in information technology with LUT. His main research interests are in color science and multispectral image processing.