# A Comparison Between the Rotation Terms Proposed by Recent CIELAB-Based Color-Difference Formulas

M. Melgosa, R. Huertas, A. Yebra, and E. Hita Departamento de Óptica. Facultad de Ciencias. Universidad de Granada Granada, España

#### Abstract

The rotation terms proposed by the BFD, LCD and CIEDE2000 color-difference formulas have been compared. These rotation terms are considered as the result of an interaction between chroma-differences and hue-differences, which doesn't exist in other recent CIELAB-based formulas, such as JPC79, CMC or CIE94. 3D plots of the rotation terms against chroma and hueangle show similar gaussian-shaped functions for LCD and CIEDE2000, while BFD propose a different function with 3 peaks at different hue angles. Deviation with respect to the radial orientation for the major axis of a\*b\* color-discrimination ellipses (tilt) is the main conesquence of a rotation term. Thus, the tilts of 151 experimental ellipses for surface colors have been compared with predictions made by the different formulas. For 31 ellipses in the blue zone  $(220^{\circ} < h < 300^{\circ})$  the best performance was found for BFD (19.6° absolute average error), followed by LCD (21.4°), CIEDE2000 (22.0°), and radial formulas such as CIE94 (31.7°). For 31 ellipses in the orange zone  $(10^{\circ} < h < 70^{\circ})$  a very similar performance was found for all the formulas. Specifically, we feel that the rotation term proposed by CIEDE2000 cannot be improved from the experimental tilts of the ellipses considered here.

#### Introduction

Recent CIELAB-based color-difference formulas such as JPC79<sup>,1</sup> CMC<sup>,2</sup> BFD,<sup>3</sup> CIE94,<sup>4</sup> LCD,<sup>5,6</sup> and CIEDE2000<sup>7</sup> follow the general pattern shown by the next equation:

$$\Delta E = \begin{bmatrix} \left(\frac{\Delta L^*}{k_L W_L}\right)^2 + \left(\frac{\Delta C_{ab}^*}{k_C W_C}\right)^2 + \left(\frac{\Delta H_{ab}^*}{k_H W_H}\right)^2 + \\ + \begin{bmatrix} R_T \left(\frac{\Delta C_{ab}^*}{k_C W_C}\right) \left(\frac{\Delta H_{ab}^*}{k_H W_H}\right) \end{bmatrix} \end{bmatrix}^{1/2}$$
(1)

where  $\Delta L^*$ ,  $\Delta C^*_{ab}$ , and  $\Delta H^*_{ab}$  are the lightness, chroma and hue-differences computed from CIELAB [8];  $W_L$ ,  $W_C$ , and  $W_H$  are designated as 'weighting function' intending to improve the perceptual uniformity of CIELAB; and  $k_L$ ,  $k_C$ , and  $k_H$  are designated as 'parametric factors' trying to account for the influence of the experimental conditions on perceived color-differences. Usually the parametric factors are kept as unity for a given set of experimental conditions, designated as 'reference conditions',<sup>4</sup> and sometimes advise is provided for deviations of such a conditions (e.g.  $k_L=2$  for textured textile samples). The last term in Equation 1 has been usually designated a 'rotation term', and is controlled by the specific  $R_T$  functions adopted by each color-difference formula. While the JPC79, CMC and CIE94 color-difference formula do not propose a rotation term (i.e.  $R_T = 0$ ), BFD, LCD and CIEDE2000 do it.

To be exact, there are a couple of differences between our previous Equation 1 and the CIEDE2000 and LCD color-difference formulas. Specifically, in the case of the CIEDE2000 formula, a re-definition of coordinate a\* leads to  $\Delta C'$  and  $\Delta H'$  different to those proposed by CIELAB; and, in the case of LCD, the rotation term is given by  $R_T (\Delta C^*_{ab}) (\Delta H^*_{ab})$ .

Chronologically, BFD was the first formula including a rotation term  $R_{_{\rm T}}$  in a CIELAB-based color-difference model, trying to account for the tilt (deviation with respect to the radial orientation) of the a\*b\* colordiscrimination ellipses in the blue region. The good results achieved by BFD for color-difference pairs in the blue region have been shown by different studies.910 In general, it is apparent that using an additional non-null rotation term in Equation 1 will lead to a better fit of visually perceived color-differences. The rotation term can be considered as the result of an interaction between chroma and hue-differences, which has been shown by different color-difference perception data sets.<sup>11,12</sup> Å potential relationship between the rotation term and the tritanopic confusion lines has been suggested by Viénot<sup>13</sup> and studied by us.<sup>14</sup>

In this paper the orientation of a color ellipse is defined as the angle between its major axis and the positive direction of the x-axis, being in the range 0°-180°. With respect to the tilt of a given ellipse, it is defined as the difference between the orientation of the ellipse and the radial orientation (the orientation of the line joining the origin and the ellipse's center), in this order. The tilt will be in the range  $\pm 90^{\circ}$ , a positive tilt indicating a counterclockwise rotation of the major axis of the ellipse with respect to the radial orientation. From our previous Equation 1 the predicted tilt  $\theta$  (in degrees) for a given color-difference formula can be computed<sup>15</sup> using the equation:

$$\theta = 28.64789 \tan^{-1} \frac{R_T W_C W_H}{W_H^2 - W_C^2}$$
(2)

Although comparisons of the weighting functions proposed by some CIELAB-based color-difference formulas have been recently reported,<sup>16,17</sup> both the rotation terms proposed by these formulas and their associated tilts have not been compared, this being the main goal of the current work.

#### **Results and Discussion**

The rotation terms  $R_{T}$  proposed by CIEDE2000, LCD and BFD, are defined as the product of a function of CIELAB chroma ( $R_c$ ), and a function of CIELAB hue-angle ( $R_{\mu}$ ). 3D plots of the rotation term for each one of these three formulas are given in Figures 1-3. As shown by Figure 1 and 2, the rotation terms of CIEDE2000 and LCD are very similar (their  $R_{H}$  functions are identical), having only non-null negative values in the blue region, which are symmetrical around the hue-angle 275°. These negative values lead to a counterclockwise (positive) tilt of the ellipses in the blue region, in agreement with experimental data.<sup>11,12</sup> For low C\* values the slope of the  $R_{T}$  function is higher for LCD than for CIEDE2000. The rotation term proposed by BFD (Figure 3) is more complex than the one adopted by CIEDE2000 and LCD, including both positive and negative  $R_{T}$  values (i.e. positive and negative tilts). For BFD, the greatest negative values of  $R_{T}$  are also in the blue region, but two additional peaks are shown in the orange and green regions.



Figure 1. 3D plot of the rotation term  $R_{\tau}$  proposed by CIEDE2000 as a function of CIELAB chroma and hue-angle.



Figure 2. 3D plot of the rotation term  $R_{\tau}$  proposed by LCD as a function of CIELAB chroma and hue-angle.



Figure 3. 3D plot of the rotation term  $R_r$  proposed by BFD as a function of CIELAB chroma and hue-angle.

As mentioned before, tilt is the main consequence of the rotation term. Figure 4 shows the experimental tilts for a set of 151 ellipses from two highly reliable data sets,<sup>11,12</sup> which have been translated to CIELAB in previous works,<sup>18,19</sup> as a function of the hue-angle of the ellipses' centers. A polynomial fit has been fitted to these data, showing that the highest positive tilts are found in the blue region, followed by the orange region. Note that negative tilts (unpredictable by the CIEDE2000 and LCD formulas) are also found for some ellipses.



Figure 4. Tilt (difference between experimental and radial orientations) of the experimental ellipses obtained by Luo et al. [11] (open symbols) and Berns et al. [12] (closed symbols) at different hue-angles.

Tilts greater than  $\pm 60^{\circ}$  in Figure 4 correspond to ellipses centers with low chroma having high values of semiaxes relationship, and their orientations become nearly meaningless.

Using the previous Equation 2, the tilts predicted by the CIEDE2000, LCD and BFD color-difference formulas have been computed for a set of points in the plane  $a^*b^*$  placed on a regular grid of 2 units. The results found are shown for each one of these formulas in Figures 5-7. As can be seen, these plots closely resemble the ones in Figures 1-3, indicating that the rotation term  $R_T$  is the main factor causing the ellipses tilt  $\theta$ .



Figure 5. Contour plots of the tilts (in degrees) predicted by the CIEDE2000 formula at different positions of the plane a\*b\*.



Figure 6. Contour plots of the tilts (in degrees) predicted by the LCD formula at different positions of the plane a\*b\*.

Table I shows the average differences between experimental and theoretical tilts predicted by different formulas: radial formulas (e.g. CIE94), CIEDE2000, LCD and BFD. The results shown in the first row of Table I correspond to the 151 experimental ellipses previously mentioned.<sup>11,12</sup> Different subsets are considered for the next rows: saturated ellipses having centers with C\*>20, ellipses in the blue region ( $220^{\circ}$ <h<300), and ellipses in the orange region ( $10^{\circ}$ <h<70^{\circ}). These subsets were selected bearing in mind that the ellipses' orientation is nearly meaningless for low saturated centers, and the two region showing the greatest tilts (see Figure 4).



Figure 7. Contour plots of the tilts (in degrees) predicted by the BFD formula at different positions of the plane a\*b\*.

Table I. Average differences (absolute value) between experimental tilt and predicted tilt by different colordifference formulas. The second column shows the number of ellipses employed in parentheses.

		Color-difference formulas			
		Radial	CIEDE 2000	LCD	BFD
0° <h<360°< td=""><td>All (151)</td><td>18.9°</td><td>17.0°</td><td>17.0°</td><td>16.2°</td></h<360°<>	All (151)	18.9°	17.0°	17.0°	16.2°
	C*>20 (109)	15.8°	13.2°	13.2°	12.5°
Blue 220° <h<300°< td=""><td>All (31)</td><td>31.7°</td><td>22.0°</td><td>21.4°</td><td>19.6°</td></h<300°<>	All (31)	31.7°	22.0°	21.4°	19.6°
	C*>20 (21)	32.0°	18.7°	18.5°	15.3°
Orange 10° <h<70°< td=""><td>All (28)</td><td>18.7°</td><td>18.7°</td><td>18.7°</td><td>17.4°</td></h<70°<>	All (28)	18.7°	18.7°	18.7°	17.4°
	C*>20 (19)	12.1°	12.1°	12.1°	10.6°

From Table I we can conclude a very similar performance of CIEDE2000 and LCD formulas, as well as a slight improvement achieved for the BFD formula, in particular for the saturated centers of the blue region. As expected, the most important improvement of predictions made by the radial formulas was also found in the blue region, in particular for the BFD formula. In any case, it should be said that, although the differences shown by Table I are greater than those attributable to usual experimental errors, they are not very high. We feel that the rotation term proposed by CIEDE2000 cannot be improved from the experimental tilts of the ellipses considered here. In any case future improvements of the recently proposed CIEDE2000 formula should also be

tested using new experimental data sets employed in the literature.  $^{\scriptscriptstyle 20}$ 

## Acknowledgements

Research Project BFM2000-1473, Ministerio de Ciencia y Tecnología (España).

### References

- 1. R. McDonald, J. Soc. Dyers Colour., 96, 486-497 (1980).
- 2. F.J.J. Clarke, R. McDonald, and B. Rigg, J. Soc. Dyers Colour., 100, 128-132 (1984).
- 3. M.R. Luo and B. Rigg, J. Soc. Dyers Colour., 103, 86-94 (1987).
- 4. CIE Publication 116-1995, Industrial color-difference evaluation, CIE Central Bureau, Vienna, 1995.
- 5. D.H. Kim, The influence of parametric effects on the appearance of small colour differences. Ph.D. Thesis, chapter 6. University of Leeds, UK; 1997.
- D.H. Kim, E.K. Cho, J.P. Kim, Col. Res. Appl., 26, 369-375 (2001)
- 7. CIE Publication 142-2001, Improvement to industrial color-difference evaluation, CIE Central Bureau, Vienna, 2001.

- CIE Publication. 15.2, Colorimetry, 2<sup>nd</sup> Ed. CIE Central Bureau, Vienna, 1986.
- M.R. Luo and B. Rigg, J. Soc. Dyers Colour., 103, 126-132 (1987).
- M. Melgosa, E. Hita, J. Romero, and L. Jiménez del Barco, J. Opt. Soc. Am. A, 9, 1247-1254 (1992).
- 11. M.R. Luo and B. Rigg, Col. Res. Appl., 11, 25-42 (1986).
- 12. R.S. Berns, D.H. Alman, L. Reniff, G.D. Snyder and M.R. Balonon-Rosen, Col Res. Appl., 16, 297-316 (1991).
- 13. Witt K. CIE TC1-47 minutes of the meeting at Warsaw 1999-06-30. Private communication.
- A. Yebra, R. Huertas, M.M. Pérez, M. Melgosa. On the relationship between the tilt of the a\*b\* tolerance ellipses in the blue region and the tritanopic confusion lines. Col. Res. Appl., Accepted, Vol. 4, 2002.
- 15. D. L. MacAdam, J. Opt. Soc. Am., 33, 18-26 (1943).
- D. Heggie, R.H. Wardman, and M.R. Luo, J. Soc. Dyers Colour., 112, 264-269 (1996).
- 17. M. Melgosa, Col Res. Appl., 25, 49-55 (2000).
- M. Melgosa, E. Hita, J. Romero and L. Jiménez del Barco, Col Res. Appl., 19, 10-18 (1994).
- 19. M. Melgosa, E. Hita, A.J. Poza, D.H. Alman and R.S. Berns, Col. Res. Appl., 22, 148-155 (1997).
- M.R. Luo, G. Cui, B. Rigg, Col. Res. Appl., 26, 340-360 (2001).