Influence of Tristimulus Values Correlation and Reference White in Error Propagation to CIELUV and CIELAB Color Parameters

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Abstract

Random errors propagation from tristimulus values to CIELUV and CIELAB color parameters has been studied considering two factors usually neglected in the literature: the correlation among tristimulus values measurements, and the associated error to the measurement of the reference white. Our study is based on experimental measurements performed by a modern spectroradiometer (SpectraScan PR-704 Photo Research), using 28 Munsell samples illuminated by 3 different luminous sources (D65, F and TL84) provided by a VeriVide color assessment cabinet. Sets of 10 measurements were made by 3 different procedures (short temporal period, large temporal period, and spatial variation), for each one of the color samples and luminous sources mentioned above. Correlation among tristimulus values reduces the error propagated to CIELUV and CIELAB color parameters in a factor up to 2.6, this effect being minimum for L* and b*. The errors in the tristimulus values of the reference white increase the error propagated to CIELUV and CIELAB color parameters in a factor up to 1.8. Anyway, under our experimental conditions, the instrumental errors of the a* and b* coordinates are lower than the visual sub-thresholds predicted by the CIE94 model in a factor about 20.

Introduction

As any other physical measurement, color coordinates specified in any quantitative system ought to be provided with their corresponding errors. Although error propagation from spectral measurements to tristimulus values and CIE x,y chromaticity coordinates has been studied in early papers,[1,2] in most practical applications color coordinates in CIELUV and CIELAB (the current color spaces recommended by the International Commission on Illumination since 1976[3]) are rarely reported with their associated errors.[4,5] The main goal of the current paper is to study random error propagation from tristimulus values to CIELUV and CIELAB color coordinates, using a given set of highly homogeneous color samples and a modern color measurement instrumentation.

In a previous paper[6] we have studied the influence of different measurement methods and light sources on error propagation from tristimulus values to CIELUV and CIELAB parameters. Now we will complete our previous results, focusing our interest on two aspects of this error propagation, which have not been analyzed in previous literature: correlation among tristimulus values measurements, and associated error to the measurement of the reference white. Bearing in mind the overlap of the CIE color-matching functions, correlation coefficients among tristimulus values should not be neglected, as reported by Gardner.[7] In addition, the reference white (included in the definitions of CIELUV and CIELAB coordinates to account for adaptation effects) is usually obtained in practical applications through direct measurements of a standard white surface, and, thus, the error associated to these measurements should also be considered. The overall errors obtained considering both correlation among tristimulus values and associated error to the reference white, will be also analyzed here from the point of view of their visual significance, using sub-threshold data[8] and predictions made by the recent CIE94 color-difference formula.[9]

Method

We selected 28 color samples from the Munsell Book of Color[10] trying to cover a wide color range, and grouped around the centers 5Y 5/4, 10G 5/4, 5PB 5/4, 5RP 5/4, following the scheme of a color tolerance set.[11] These samples were positioned in a VeriVide color assessment cabinet and were illuminated by 3 of its light sources (D65, F and TL84). Color measurements of each color sample under each light source were made by a SpectraScan PR-704 spectroradiometer, following 3 procedures: consecutive temporal measurements (short temporal period), measurements at 10-minutes intervals (large temporal period), and measurements at different points of the sample (spatial measurement). The short temporal period is perhaps the most usual procedure employed in practical applications, but the other two procedures should also be useful, for example, in the cases of unstable or heterogeneous samples. Ten measurements of the tristimulus values were made using each one of these three procedures. The average value of these 10 measurements was considered the value of the magnitude, and the standard deviation the associated error. A barium-sulphate-powder plaque, provided with our spectroradiometer, was also measured by these 3 procedures, and the results obtained were used as the appropriate reference white for transformations to

CIELUV and CIELAB.[3] Correlation coefficients among tristimulus values were computed from spectral data using the method proposed by Gardner.[7]

Cuadratic error propagation theory[12] is usually employed when it can be assumed that the main errors in direct measurements are random, as in our current work. From this theory, for the general case of a magnitude V depending of only two magnitudes α and β, which cannot not be considered independent, the variance of magnitude V is obtained from the equation:

$$\sigma^2(V) = \left(\frac{\partial V}{\partial \alpha}\right)^2 \sigma^2(\alpha) + \left(\frac{\partial V}{\partial \beta}\right)^2 \sigma^2(\beta) + 2r_{\alpha\beta}\left(\frac{\partial V}{\partial \alpha}\right)\left(\frac{\partial V}{\partial \beta}\right)\sigma(\alpha)\sigma(\beta) \quad (1)$$

where $r_{\alpha\beta}$ indicates the non-null correlation coefficient between α and β, and σ represents the standard deviation. In our case, the magnitude V corresponds to each one of the CIELUV and CIELAB parameters (that is, u*, v*, L*, a*, b*), and Equation 1 leads to different functions of the tristimulus values of both the Munsell sample and the reference white.[13]

Results and Discussion

Tables I and II show the average standard deviation of the tristimulus values measured using the three light sources and the three procedures described above. Specifically Table I shows the average standard deviation obtained for the 28 Munsell samples and Table II the standard deviation of the measurements made for the reference white. As we can see, the lowest errors are obtained for the short temporal period procedure for the three light sources. The errors for large temporal period and spatial measurement are similar, and one order of magnitude higher than those for short temporal period, approximately. It should also be noted that measurements made under the source F lead to the greatest errors, for the three procedures of measurement employed, suggesting a low stability of this luminous source. It is a little surprising that errors in Table I are greater than those in Table II, when it could be assumed that our standard white surface is more homogeneous than the Munsell samples. This should be explained because Tables I and II become from standard deviations of tristimulus values, which usually increase with the tristimulus values, and are lower for the Munsell samples than for the reference white.

Table I. Average over 28 Munsell samples of the standard deviation of 10 measurements of the tristimulus values made with 3 light sources (D65,F,TL84) and 3 procedures (short temporal period, large temporal period, spatial measurement).

Source	Procedure	X	Y	Z
D65	Short Temp. Period	0.006	0.005	0.009
	Large Temp. Period	0.04	0.04	0.06
	Spatial Measurement	0.05	0.05	0.05
F	Short Temp. Period	0.16	0.13	0.03
	Large Temp. Period	1.2	1.0	0.2
	Spatial Measurement	0.8	0.7	0.13
TL84	Short Temp. Period	0.007	0.009	0.005
	Large Temp. Period	0.05	0.05	0.04
	Spatial Measurement	0.10	0.10	0.06

Table II. Standard deviation of 10 measurements of the tristimulus values of the reference white, made with 3 light sources (D65,F,TL84) and 3 procedures (short temporal period, large temporal period, spatial measurement).

Source	**Procedure**	**Xo**	**Yo**	**Zo**
D65	Short Temp. Period	0.03	0.02	0.04
	Large Temp. Period	0.3	0.3	0.5
	Spatial Measurement	0.18	0.17	0.3
F	Short Temp. Period	1.0	0.8	0.18
	Large Temp. Period	2	2	0.4
	Spatial Measurement	0.6	0.5	0.12
TL84	Short Temp. Period	0.05	0.05	0.03
	Large Temp. Period	0.19	0.19	0.16
	Spatial Measurement	0.2	0.2	0.2

Table III. Average values of the correlation coefficients among tristimulus values of our 28 Munsell samples, for the three luminous sources employed

Source	r_{XY}	r_{YZ}	r_{XZ}
D65	0.74943	0.08602	0.28886
F	0.85798	0.07815	0.03816
TL84	0.75929	0.05557	0.11105

Table III shows the average values for the 28 Munsell samples of the three correlation coefficients among tristimulus values, for each one of the three luminous sources employed. These coefficients were computed from spectral data using Gardner's procedure.[7] As expected from the different overlap of the three color-matching functions, the greatest coefficient is r_{XY} for the three sources, and the lowest one r_{YZ} (except for the F source, where $r_{XZ}<r_{YZ}$ because of the low radiance of this source at the short wavelengths, which modulates the overlap between the x and z color-matching functions).

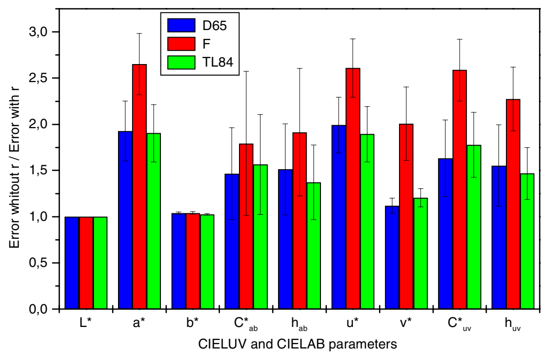

Figure 1. Relationship of the errors found for different CIELUV and CIELAB parameters without and with considering the correlation coefficients among tristimulus values. The average results of the 28 Munsell samples are shown, together with their respective standard deviations (error bars). The short temporal period measurements have been used for each one of the luminous sources employed.

Figure 1 shows the influence of correlation coefficients on the error propagated to CIELUV and CIELAB color parameters, for each one of the luminous sources employed and the measurements made by short temporal period. As can be seen, correlation among tri-stimulus values reduces the error propagated to CIELUV and CIELAB color parameters in a factor up to 2.6 (except for L* where there are no correlation coefficients). The main reason for this reduction is that, in our case, the crossed terms multiplying the correlation coefficients (see Equation 1) are negative, except one of the terms found in u* and v*. The influence of correlation coefficients on a* is greater than on b*, because the r_{XY} coefficient involved in the definition of a* is greater than the r_{YZ} coefficient involved in b* (see Table III). Figure 1 also shows that the influence of correlation coefficients is similar for the sources D65 and TL84, and lower for the source F, because for this last source the errors associated to tristimulus values are greater, as shown in Tables I and II.

Figure 2 shows the results found when the reference white is considered without / with its associated error, for the source D65 and each one of the three procedures of measurement. We can see that including the error associated to the reference white increases the corresponding errors of CIELUV and CIELAB parameters in an up to 1.8 factor. This is not a surprising result, because the number of terms in the error propagation equation (see Equation 1) increases when the errors in the experimental measurements of the reference white are considered. In particular, it is also reasonable that for coordinate L* the influence of the error associated to the reference white is lowest, because this coordinate depends only of one of the tristimulus values of the reference white.

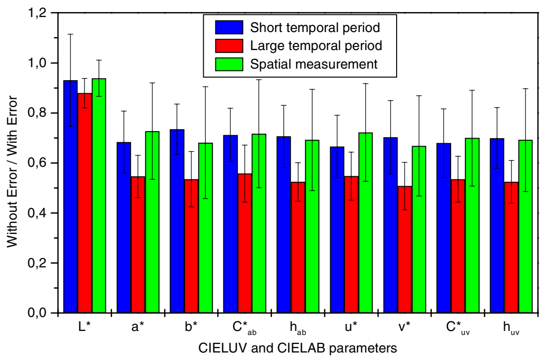

Figure 2. Relationship of the errors found for different CIELUV and CIELAB color parameters without and with considering the error associated to the experimental measurements of the reference white. The results plotted correspond to the D65 source, and the average of the 28 Munsell samples with their respective standard deviations (error bars).

Eventually, Figure 3 shows a comparison of the computed errors in the a*,b* CIELAB coordinates for our 4 main Munsell samples (centers 5Y 5/4, 10G 5/4, 5PB 5/4, 5RP 5/4) with visual sub-thresholds predicted by the CIE94 color difference model [9]. Similar results can be found for the other CIELAB or CIELUV coordinates. The effect of both correlation coefficients among tristimulus values and error associated to the reference white have been considered in computations leading to the errors of the a* and b* coordinates shown in Figure 3. The color difference assumed for the ellipses plotted in Figure 3 is 0.23 CIE94 units, which could be considered a sub-threshold color difference.[8] Note that, for an easier viewing, the errors of the a* and b* coordinates are represented in Figure 3 by a cross 10 times the true value. Fortunately, we can conclude that the instrumental errors found by us for the a* and b* coordinates could not be perceived by observers with normal color vision, as desired. Anyway, we should remember that the results plotted in Figure 3 were obtained using D65 source, short temporal period procedure of measurement, homogeneous samples and a modern color instrumentation. Errors found for other different setups should be higher than our current ones.

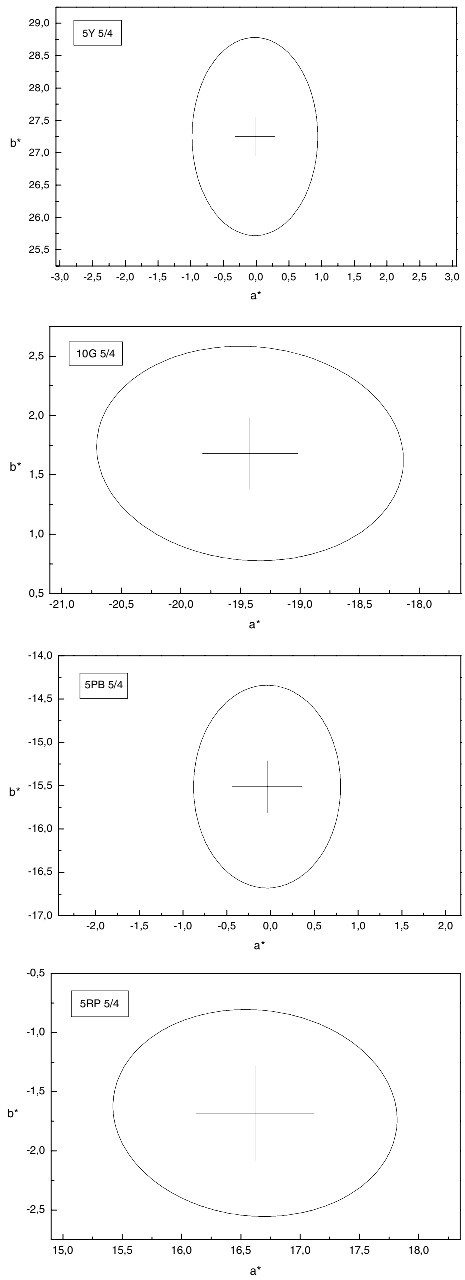

Figure 3. Errors in a and b* CIELAB coordinates (increased 10 times) compared with the sub-threshold ellipses predicted by CIE94 (0.23 CIE94 units) for 4 Munsell samples. The errors of the a* and b* coordinates were computed considering both correlation among tristimulus values and reference white errors. D65 source and measurements by short temporal period have been assumed.*

Conclusions

In summary, assuming that tristimulus values are independent the corresponding errors of CIELUV and CIELAB color parameters are over-estimated in an up to 2.6 factor, while considering the reference white without any error under-estimates the errors of CIELUV and CIELAB parameters in an up to 1.8 factor. In our opinion, both correlation coefficients among tristimulus values and associated error to the reference white should not be neglected in practical applications seeking the true errors associated to CIELUV and CIELAB color parameters. In our current work these errors are lower than visual sub-thresholds.

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References

1. I. Nimeroff, J. Opt. Soc. Amer., 43, 531-533 (1953).
2. A. R. Robertson, J. Opt. Soc. Amer., 57, 691-698 (1967).
3. CIE Publication. 15.2, Colorimetry, 2nd Ed. CIE Central Bureau, Vienna, 1986.
4. M. D. Fairchild and L. Reniff, Col. Res. Appl., 16, 360-367 (1991).
5. P. D. Burns and R. S. Berns, Col. Res. Appl., 22, 280-289 (1997).
6. R. Huertas, M. Melgosa, and E. Hita, Error in CIELAB and CIELUV color coordinates for Munsell samples under different light sources,. Proc. SPIE Vol. 4419, pg. 585-588, (2001).
7. J. L. Gardner and R. B. Frendel, Metrologia, 36, 477-480 (1999).
8. K. Witt, Col. Res. Appl., 15, 189-199 (1990).
9. CIE Publication 116-1995, Industrial color-difference evaluation, CIE Central Bureau, Vienna, 1995.
10. Munsell Book of Color, Glossy Finish Collection, Macbeth Kollmorgen, 1976.
11. ASTM standards on color and appearance measurement, Fifth Edition, 1996. Standard terminology of appearance (E 284), pg. 240.
12. P. R. Bevington and D. K. Robinson, Data reduction and error analysis for the physical sciences, 2nd Ed., McGraw-Hill, 1992.
13. R. Huertas, Estudio sobre el error asociado a los parámetros de color CIELAB y CIELUV. MD Dissertation, Universidad de Granada, 2001.