Adaptive Varying Window Size Filtering of Color Images Based on Intersection of Confidence Intervals Rule

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Abstract

A class of linear/nonlinear filters with varying adaptive window size is studied. After some (optional) color transformation, for each of three color channels the window size of the applied mask-filters is considered as a parameter. The intersection of confidence intervals (ICI) rule is used for selection of the adaptive window size based on the filter's outputs obtained for different window sizes. In parallel five filters with symmetric and four quadrants masks are used. The ICI rule gives the adaptive window sizes for each of these filters and in a point-wise manner for each pixel of the image. This adaptive window size filters are able to suppress the noise efficiently provided that color edges are well preserved. The final filtering output is obtained by combining outputs of the mentioned five partial filters, each with the varying adaptive window sizes. This operation is produced for each color channel. Finally, we convert the estimates of the color image components back to RGB image.

Originally, the ICI rule has been proved by theoretical and empirical studies to be efficient for linear and median filters. We show how this ICI rule can be modified and applied for color image filtering.

Simulation experiments confirm that the ICI rule used for window size selection of the mean and median filters of the multichannel combined filters is able to significantly improve quality of color image filtering. The performance of the filters is characterized both by the accuracy and human visual perception criteria.

1. Introduction

Linear filters have long been used in many signal and image processing applications. They can be easily analyzed and implemented. While linear filters can be optimal when a signal or image is corrupted by Gaussian additive noise, other noise contamination situations, such as impulsive, speckle, or signal dependent noise, demand the use of nonlinear filters.[1,3] Several important nonlinear filter classes that have gained much popularity are classes based on monotone (positive) Boolean functions. This class includes median and related filters, weighted order statistic filters, and morphological filters with .at structuring elements.[3]

It is well known that filters with fixed parameters are not as effective as adaptive filters, whose parameters depend on the local behavior of the signal, which is assumed to be nonstationary, as is the case for images.[1] One important parameter is the window size of the filter. For example, for the mean filter smaller window sizes lead to better detail preservation, but worse noise attenuation capability than larger window sizes. Thus, window size selection is one of the key questions in a filter design. This trade-off between detail preservation and noise suppression takes place in many situations. In statistical terms, it is a trade-off between the bias and variance of estimation.

In this paper we present an algorithm of spacevarying filtering based on a recently developed method, called the Intersection of Confidence Intervals (ICI) rule.[4] Originally, this rule was proposed and justified for local polynomial approximation (LPA) scalar linear filters. A generali-zation to two dimensional signals is proved to be efficient for gray-level image de-noising. In particular, in Ref. 5 and Ref. 7, the image de-noising problem is considered for quite a general observation model including image dependent noise. The de-noising and window size selection are combined into one algorithm where the adaptive windows and signal de-noising are produced on the basis of the same observation model. A development of the ICI rule for the median filters is given in Ref. 6.

The main original results of this paper are concerned with the development of the following algorithm. First, an RGB image is transformed in the other color space (e.g., Opponent, CIELAB, Munsell, etc.). Then, for each of three channels of the new model the window size of the applied mask-filters is considered as a parameter. Lets us call this mask-filters by elementary ones. The ICI rule is used for selection of the adaptive window size based on the filter's outputs obtained for different window sizes. In parallel five filters with symmetric and four quadrants masks are used. The ICI rule gives the adaptive window sizes for each of these filters and in a point-wise manner for each pixel of the image. This adaptive window size filters are able to suppress the noise efficiently provided that color edges are well preserved. The final filtering output is obtained by combining outputs of the mentioned five partial filters, each with the varying adaptive window sizes. This operation is produced for each of the color channels. Finally, we convert the estimates of the color image components back to RGB image.

In this paper we restrict a class of the applied elementary filters to simple mean and median, which results in an efficient implementation in a general quite complex multichannel filters. Simulation experiments confirm that the ICI rule used for window size selection of the mean and median filters of the multichannel combined filters is able to significantly improve quality of color image filtering. The performance of the filters is characterized both by the accuracy and human visual perception criteria.

2. Combined Weighted Mean/Median Filters

Let $Z^{(j)}(k,l)$, with (k,l) integers, j = 1, 2, 3, be noisy observations of a color ($M \times N \times 3$) image in any given color space (linearly or nonlinearly related to the RGB 2) obtained as a contamination of a true image $A^{(j)}(k,l)$ by an additive noise $n^{(j)}(k,l)$ as follows:

$$Z^{(j)}(k,l) = A^{(j)}(k,l) + n^{(j)}(k,l), \tag{1}$$

$$0 \le k \le M - 1, 0 \le l \le N - 1, j = 1, 2, 3,$$

where the $n^{(j)}(k,l)$ are i.i.d. random errors with $E(n^{(j)}(k,l)) = 0$ and $E(n^{(j)}(k,l))^2 = \sigma^2$.

Let $W_{h,q}$ denote five masks - local neighborhoods of the pixel (k,l) in $Z^{(j)}(k,l)$, where h is a size of the mask, and q is a parameter defining a position of these windows respective to the pixel (k,l) (see Figure 1 for graphical interpretation).

We define a 3-channel combined weighted mean/median filter for each of the color plans in the following manner. First, independently, for each color plans, and for each of five windows $W_{h,q}$, $0 \le q \le 4$ we aim to obtain the "best" estimate of the current pixel $\hat{A}^{(j)}_{h,q}(k,l)$. Next, we like to obtain the "best" point-wise combination of the five obtained estimates $\hat{A}^{(j)}_{h,q}(k,l)$ for each pixel to get a final estimate $\hat{A}^{(j)}_{h}(k,l)$ for the $(k,l)th$ pixel in the *jth* color plan.

This filtering procedure has the following sequential steps:

(1) The estimates for each of five masks are found for window sizes $h \in H$, where H is a given set of windows:

$$\hat{A}^{(j)}_{h,q}(k,l) = \arg\min_m \left(\sum_{(s,t)\in W_{h,q}} \rho_h(s,t) F_1(Z^{(j)}(k+s,l+t) - m) \right) \tag{2}$$

(2) The *ICI* rule is used in order to find the optimal window sizes $h^+ \in H$ for these estimates,

(3) The estimates with the optimal window sizes are combined and this combined estimate is a solution of the optimization problem

$$\hat{A}^{(j)}(k,l) = \arg\min_n \left(\sum_{q=0}^{4} \rho_q F_2(\hat{A}^{(j)}_{h^*,q}(k,l) - n) \right)$$
$$0 \le k \le M-1,\ 0 \le l \le N-1,\ j = 1,2,3. \tag{3}$$

The weights in (2) and (3) are nonnegative and normalized: $\rho_q \ge 0, \sum_{q=0}^{4} \rho_q = 1$ and $\rho_h(s,t) \ge 0$, $\sum_{(s,t)\in W_{h,q}} \rho_h(s,t) = 1$, $\rho_h(0,0) = \max_{s,t} \rho_h(s,t)$. The weight $\rho_h(s,t)$ is of the form: $\rho_h(s, t) = \rho(s/h, t/h)/h^2$, where the parameter h defines the window size (or filter's bandwidth) and controls the scale of the estimation residuals with respect to both variables s and t. The 2D mask $\rho_h(s,t)$, with $(s,t) \in W_{h,q}$ gives the weights applied to every observation inside the local neighborhood of $Z^{(j)}(k,l)$. In the simple constant weight case: $\rho_q = 1/5$ for all q, and $\rho(s,t) = 1$, $(s, t) \in W_{h,q}$, for five 2*D* square central (q = 0) and quadrant masks $1 \le q \le 4$ (see Figure 1).

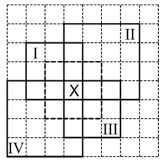

Figure 1. Graphical interpretation of the masks: X *corresponds to the pixel (*k,l*),* I, II, III, IV *show the quadrant masks (*q = *1, 2, 3, 4),* q = *0corresp onds to the mask centered with respect to* X.

The *loss functions* $F_1(x)$, $F_2(x)$ used in this paper are particularized to $|x|$ or x^2. Then, all of the the estimates in (2) and (3) are the weighted means and medians. Due to the constant weights in the used masks, the weighted mean/median filter in (2)-(3) are eventually simplified to the following four combined mean/median filters:

mean - mean filter: $F_1(x) = F_2(x) = x^2$,

mean - median filter : $F_1(x) = x_2$, $F_1(x) = |x|$,

median - mean filter : $F_1(x) = |x|$, $F_2(x) = x_2$,

median - median filter : $F_1(x) = F_2(x) = |x|$.

The outputs of these filters are defined as solutions of the following special case of (2)-(3):

$$\hat{A}^{(j)}_{h,q}(k,l) = \arg\min_m \left(\sum_{(s,t)\in W_{h,q}} F_2(Z^{(j)}(k+s,l+t) - m) \right)$$

and

$$\hat{A}^{(j)}(k,l) = \arg\min_n \left(\sum_{q=0}^{4} F_1(\hat{A}^{(j)}_{h^+,q}(k,l) - n) \right)$$

3. The ICI Rule For Weighted Mean/Median Filters

Let us introduce a finite set of window sizes $H = \{h_r : h_1 < h_2 < .... < h_J\}$, starting with a small h_1 and the corresponding estimates $\hat{A}^{(j)}_{h,q}(k,l)$ of the true signal $A^{(j)}(k,l)$ obtained with the window h^* Let $\omega_h(k,l)$ and $\sigma_h(k,l)$ be the bias and the standard deviation of this estimate. Denote by $h^*(k,l)$ the ideal window size corresponding to the minimum value of the mean squared error $E\left\{\left(\hat{A}^{(j)}_{h,q}(k,l) - A^{(j)}(k,l)\right)^2\right\}$. For quite general classes of the filters and signals the asymptotic estimation error has the following properties:

(1) For the ideal window size the ratio $|\omega_{h^*(k,l)}(k,l)|/\sigma_{h^*(kl)}(k,l) = \gamma > 0$ is constant independent of the signal;

(2) The bias is smaller than the standard deviation, $|\omega_h(k,l)| < \gamma\sigma_h(k,l)$, for $h < h^*(k,l)$ and the bias is larger than the standard deviation, $|\omega_h(k,l)| > \gamma\sigma_h(k,l)$, for $h > h^*(k,l)$.

Then the $h^*(k,l)$ gives the optimal bias-variance balance and the estimate of this $h^*(k,l)$ can be obtained as follows. We determine a sequence of confidence intervals $\mathcal{D}(r)$ of the biased estimate $\hat{A}^{(j)}_{h,q}(k,l)$, where $h = h_r$, as follows

$$D(r) = \left[\hat{A}^{(j)}_{h_r,q}(k,l) - \Gamma\sigma_{h_r}(k,l), \hat{A}^{(j)}_{h_r,q}(k,l) + \Gamma\sigma_{h_r}(k,l)\right], \quad r = 1,...,J, \tag{4}$$

where Γ is a threshold parameter of the confidence interval depending on γ and the probability that the signal $A^{(j)}(k,l)$ belongs to $\mathcal{D}(r)$. In this paper we treat Γ as a design parameter of the algorithm.

The following describes the ICI rule that is used in order to obtain the adaptive window size[4]:

Consider the intersection of the intervals $\mathcal{D}(r)$, $1 \le r \le i$, with increasing i, and let i^+ be the largest of those i for which the intervals $\mathcal{D}(r)$, $1 \le r \le i$, have a point in common. This i^+ defines the adaptive window size h(k,l) and the adaptive estimate as follows*

$$\hat{A}^{(j)}_q(k,l) = \hat{A}^{(j)}_{h^*(k,l),q}(k,l). \tag{5}$$

Thus, the ICI rule gives both the optimal estimate and the corresponding adaptive window size.

This algorithm can be justified, at least in the asymptotic sense with quite general assumptions and for a rather general class of estimates. It is emphasized that for the implementation of the ICI rule we need the estimates $\hat{A}^{(j)}_{h,q}(k,l)$ of the signal and the corresponding standard deviations $\sigma_h(k,l)$ obtained for different window sizes. For the mask-filters with the constant weights $\sigma^2_h(k,l) = V\sigma^2/\#$, where # is a number of pixels in the window, $V = 1$ or $\pi/2$ for the mean and median estimates respectively. The estimate of the standard deviation of the noise σ is obtained as a median of finite differences of the noisy image for each of the color channels independently according to the formula $\hat{\sigma} = \{median(|Z^{(j)}(k,l) - Z^{(j)}(k+p,l+q)| : (p,q) = (0,1), (1,0);\ \forall\, k,l\}/(\sqrt{2}\cdot 0.6745)$.

4. Simulations

Four RGB color images: "Lenna" and "Peppers" of size (480 × 512 × 3), "Fruits" (512 × 512 × 3) and "House" (256 × 256 × 3), shown in Figure 2, are test images in our experiments.

Optimal window sizes for a first color plane for the image "Peppers" in the case when no noise is added are shown in Figure 3 for all 5 assigned neighborhood masks in order to demonstrate window size selection using ICI rule.

The set of window sizes in these experiments was

$$h = \{1, 2, 3, 5\}.$$

One can clearly see that ICI algorithm detects edges accurately by assigning smaller window size to edge points and larger window sizes to non-edge pixels. Optimal window sizes applying ICI rule to noisy "Peppers" image (additive zero mean Gaussian noise with variance 400 in all channels) shown in Figure 4, demonstrate robustness of the ICI algorithm. Note that here we see edges much thinner than in the previous image, thus providing better filtering ability in the case of noise present.

Several noisy observations of all 4 test images from Figure 2 are obtained by adding to each color channel of these images Gaussian noise with zero mean and variances varying from 0 to 900.

Figure 2. The test images

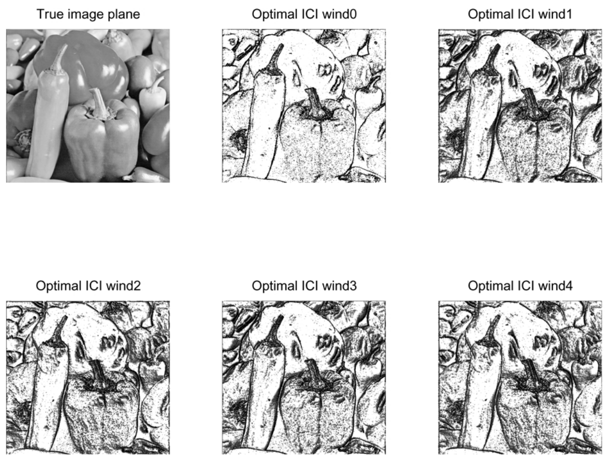

Figure 3. The first color plane of the true image "Peppers" and 5 optimum window sizes detected by ICI rule

For our experiments we have chosen the opponent color space as one of the physiologically motivated color spaces, and due to the fact that three color features constitute an effective set of features for segmenting color images.[2] This is one of the simplest (from computational point of view) linear transforms of the color RGB space, given by:

$$O1 = \frac{R+G+B}{3},$$
$$O2 = R - B,$$
$$O3 = \frac{2G - R - B}{2}.$$

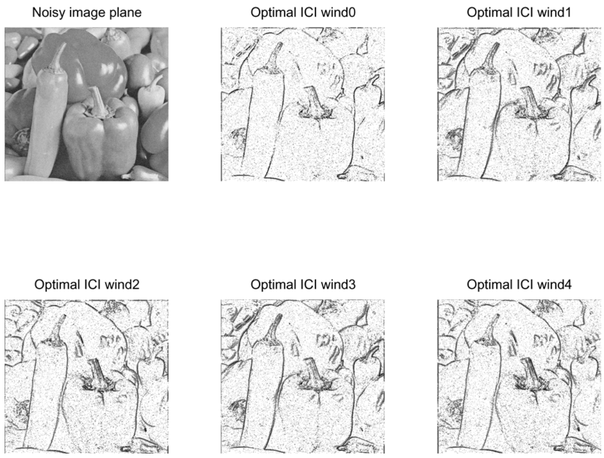

Figure 4. The first color plane of the noisy image "Peppers" and 5 optimum window sizes detected by ICI rule

The results of experiments with varying window size Mean-Mean-ICI filters on noisy (Gaussian noise) test images are presented in Figure 5 in comparison with the corresponding results of fixed size Mean filters. In this Figure we have 4 subplots, showing average error measures (of all 4 images) presenting PSNR (peak signal-to-noise ratio), MSE (mean square error), MAE (mean absolute error) and MaxDif (Maximum difference), respectively, between filtered images and noise-free original images. These error measures are defined as follows:

$$MSE(X,Y) = \frac{1}{3MN}\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{3}\left[X(i,j,k) - Y(i,j,k)\right]^2;$$

$$PSNR(X,Y) = 10 * \log_2\left(\frac{255^2}{MSE(X,Y)}\right);$$

$$MAE(X,Y) = \frac{1}{3MN}\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{3} abs\left[X(i,j,k) - Y(i,j,k)\right];$$

$$MaxDif(X,Y) = \max(\max(\max(abs(X-Y)))).$$

Error curves in Figure 5 correspond to the following cases: no filtered (dotted), filtered by mean (3 × 3) filter (dashdot), by mean (5 × 5) filter (dashed) and by our Mean-Mean-ICI varying window size filter (solid).

For the next set of experiments we have several noisy observations of 4 test images from Figure 2, obtained by adding to each color channel of these images noise with Laplacian distribution of variances varying from 0 to 900.

Again as in the previous set of experiments, we start by demonstrating ability of ICI rule to properly switch between different window sizes. Figure 6 shows optimal windows for a first color plane for the image "House" in the case when no noise is added.

The set of window sizes in this set of experiments was

$$h = \{2, 3, 5, 7\}.$$

In Figure 7 we show optimal windows selected using ICI rule for noisy "House" image (additive zero mean Laplacian noise with variance 400 in all channels).

The results of experiments with varying window size Median-Median-ICI filters are presented in Figure 8 in comparison with the corresponding results of fixed size Median filters applied separately to each color channel. Again, as in Figure 5, we show 4 subplots, with average error measures (of all 4 images) presenting PSNR, MSE, MAE and MaxDif, respectively, between filtered images and noise-free original images. Error curves correspond to the following cases: no filtered (dotted), filtered by median (3 × 3) filter (dashdot), by median (5×5) filter (dashed) and by Median-Median-ICI varying window size filter (solid).

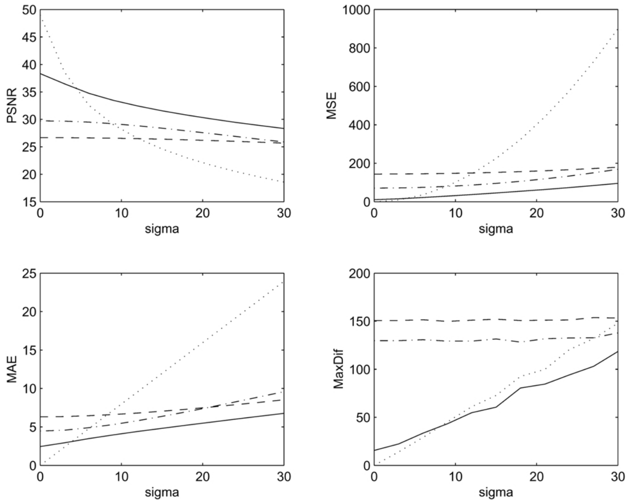

Figure 5. Average (over 4 test images) error measures (PSNR, MSE, MAE and MaxDif) versus noise sigma (standard deviation of the additive Gaussian noise) for non-filtered images, filtered by mean (3-by-3), (5-by-5), and Mean-Mean-ICI filters

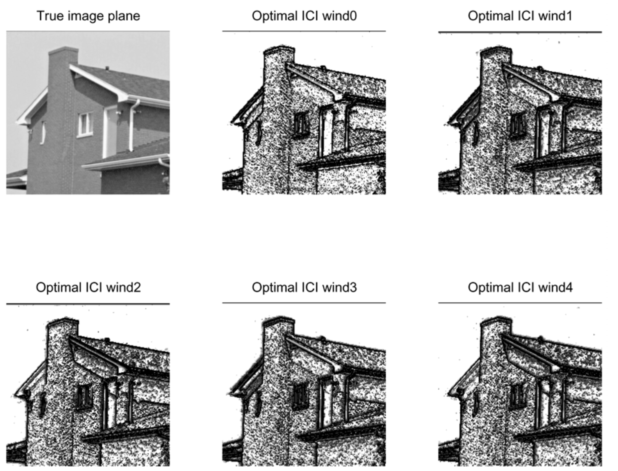

Figure 6. The first color plane of the true image "House" and 5 optimum window sizes detected by ICI rule

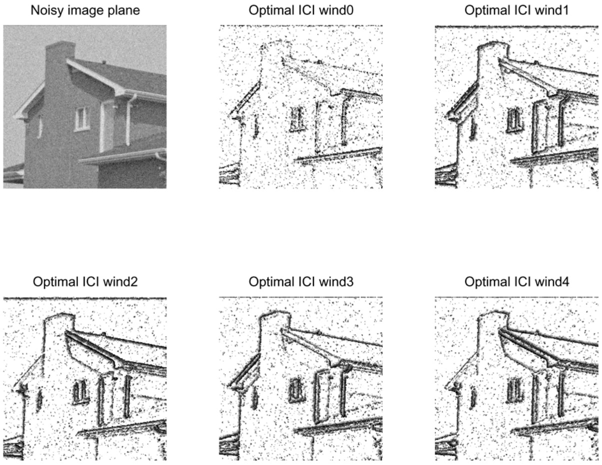

Figure 7. The first color plane of the noisy image "House" and 5 optimum window sizes detected by ICI rule

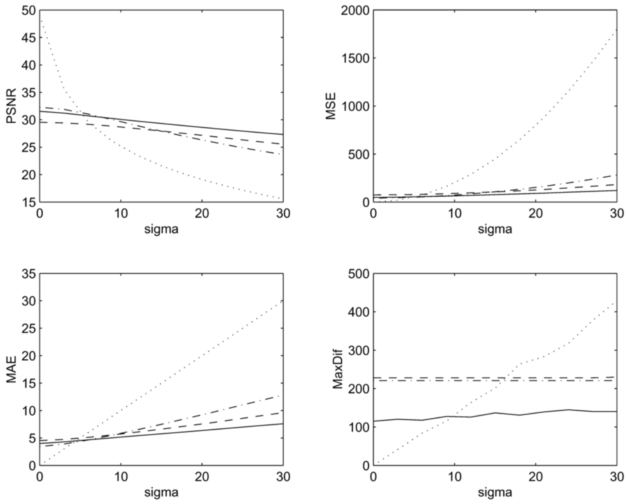

Figure 8. Average (over 4 test images) error measures (PSNR, MSE, MAE and MaxDif) versus noise sigma (standard deviation of the additive Laplacian noise) for non-filtered images, filtered by median (3-by-3), (5-by-5), and Median-Median-ICI filters

5. Conclusions

This paper is introduced adaptive window size algorithms for filtering color image data. We consider in parallel the mean/median filters with square symmetric and non-symmetric windows and determine the filter output as a combination of the outputs of these mean/median filters. The window size of the mean/median filters is varying and adaptive to unknown color signals. The *ICI* rule is developed for data-driven window size selection. The algorithm is efficient and simple to implement. It requires calculation of the mean/median estimates and their standard deviations for a set of the window size values. The threshold parameter Γ in (4) plays an important role in the performance of the algorithm. Too large or too small Γ results in over-smoothing or under-smoothing data, respectively. The parameter Γ is treated as a designed parameter of the algorithm, fixed or data-driven estimated by some statistical methods. In this paper we consider the fixed value $\Gamma = 1.5$

The behavior of the adaptive filters is analysed and their performance is compared with that of the most commonly used nonadaptive mean/median filters with square symmetric nonvarying size masks.

6. Acknowledgements

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Biography

Karen Egiazarian was born in Yerevan, Armenia, in 1959. He received the M.Sc. degree in mathematics from Yerevan State University in 1981, and the Ph.D. degree in Physics and Mathematics from Moscow M.V. Lomonosov State University in 1986. In 1994 he was awarded the degree of Doctor of Technology by Tampere University of Technology, Finland. He has been a Senior Researcher at the Department of Digital Signal Processing of the Institute of Information Problems and Automation, National Academy of Sciences of Armenia. He is currently a Full Professor in the Department of Signal Processing at Tampere University if Technology. His research interests are in the areas of applied mathematics, digital logic, signal and image processing. He has published more than 200 articles in these areas, and is coauthor (with S. Agaian and J. Astola) of the book "Binary Polynomial Transforms and Nonlinear Digital Filters", published by Marcel Dekker, Inc. in 1995, and three book chapters.